# Question A (10 marks) - Credit Card Mathematics

Introduction

On a monthly credit card balance of $1000, a typical credit card company will only ask for a minimum payment of $20. Why do credit card companies do that?

Mathematics of Credit Card Debt

Suppose we do what the company wants and make only the minimum payment $p$ every month against an initial balance of $b$. If the company charges monthly interest rate $r$, what is the balance after $n$ months?

See if we can notice a pattern.

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| --- |
| Balance after $n$ months |
| $$n=1$$ | $$\left(b-p\right)\left(1+r\right)=b\left(1+r\right)-p\left(1+r\right)$$ |
| $$n=2$$ | $$b\left(1+r\right)^{2}-p\left(1+r\right)^{2}-p\left(1+r\right)$$ |
| $$n=3$$ | $$b\left(1+r\right)^{3}-p\left(1+r\right)^{3}-p\left(1+r\right)^{2}-p\left(1+r\right)$$ |
| $$n=4$$ | $$b\left(1+r\right)^{4}-p\left(1+r\right)^{4}-p\left(1+r\right)^{3}-p\left(1+r\right)^{2}-p\left(1+r\right)$$ |

A1. (2 marks) Looking at the pattern above, derive a general function, $f\left(n,r,p,b\right)$, for the balance after $n$ months. *Hint: use summation notation* $\sum\_{k=1}^{n}$ *where applicable when deriving the function.*

A2. (1 mark) If your credit card company charges a monthly interest rate of 2% (annually 24%) on an initial balance of $1000, and you make a monthly payment of $30, what is your balance after one year? That is, find the value of $f\left(12,0.02,\$30,\$1000\right)$.

A3. (1 marks) Based on your answer in A2, how much did you end up paying in interest rate charges over a year?

A4. (2 marks) Use geometric progression properties to convert the general formula in A1 above to a functional form that excludes the summation notation. *Hint: You want to replace the summation notation* $\sum\_{i=1}^{n}$ *with a ratio; see* [*https://en.wikipedia.org/wiki/Geometric\_progression*](https://en.wikipedia.org/wiki/Geometric_progression)*, subsection titled Related Formulas.*

A5. (2 marks) How many months would it take to pay off a balance of $1000 if you made $30 monthly payments while being charged 2% monthly interest? What if we double the payment to $60, do we cut the time in half? *Hint: equate the function for the balance after* $n$ *month to zero and solve for* $n$*.*

A6. (2 marks) Plot the function derived in A5 in a two-dimensional coordinate system with $n$ on the $y$-axis and $p$ on the $x$-axis. Assume the initial balance of $b=\$1000$, and monthly interest of $r=0.02$. Find the vertical asymptote of this function, that is, find the value $p$ (monthly minimum payment on your credit card) such that the number of months required to pay off your credit card debt is equals to infinity (that is a monthly minimum payment that makes you forever indebted to your credit card provider!).