# Question A (10 marks) - Credit Card Mathematics

Introduction

On a monthly credit card balance of $1000, a typical credit card company will only ask for a minimum payment of $20. Why do credit card companies do that?

Mathematics of Credit Card Debt

Suppose we do what the company wants and make only the minimum payment every month against an initial balance of . If the company charges monthly interest rate , what is the balance after months?

See if we can notice a pattern.

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| --- | --- |
| Balance after months | |
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A1. (2 marks) Looking at the pattern above, derive a general function, , for the balance after months. *Hint: use summation notation where applicable when deriving the function.*

A2. (1 mark) If your credit card company charges a monthly interest rate of 2% (annually 24%) on an initial balance of $1000, and you make a monthly payment of $30, what is your balance after one year? That is, find the value of .

A3. (1 marks) Based on your answer in A2, how much did you end up paying in interest rate charges over a year?

A4. (2 marks) Use geometric progression properties to convert the general formula in A1 above to a functional form that excludes the summation notation. *Hint: You want to replace the summation notation with a ratio; see* [*https://en.wikipedia.org/wiki/Geometric\_progression*](https://en.wikipedia.org/wiki/Geometric_progression)*, subsection titled Related Formulas.*

A5. (2 marks) How many months would it take to pay off a balance of $1000 if you made $30 monthly payments while being charged 2% monthly interest? What if we double the payment to $60, do we cut the time in half? *Hint: equate the function for the balance after month to zero and solve for .*

A6. (2 marks) Plot the function derived in A5 in a two-dimensional coordinate system with on the -axis and on the -axis. Assume the initial balance of , and monthly interest of . Find the vertical asymptote of this function, that is, find the value (monthly minimum payment on your credit card) such that the number of months required to pay off your credit card debt is equals to infinity (that is a monthly minimum payment that makes you forever indebted to your credit card provider!).