CHAPTER 5

Forecasting

To accompany
Quantitative Analysis for Management, *Twelfth Edition*,
by Render, Stair, Hanna and Hale
Power Point slides created by Jeff Heyl

LEARNING OBJECTIVES

After completing this chapter, students will be able to:

1. Understand and know when to use various families of forecasting models.
2. Compare moving averages, exponential smoothing, and other time-series models.
3. Seasonally adjust data.
4. Understand Delphi and other qualitative decision-making approaches.
5. Compute a variety of error measures.

CHAPTER OUTLINE

5.1 Introduction

5.2 Types of Forecasting Models

5.3 Components of a Time Series

5.4 Measures of Forecast Accuracy

5.5 Forecasting Models – Random Variations Only

5.6 Forecasting Models – Trend and Random Variations

5.7 Adjusting for Seasonal Variations

5.8 Forecasting Models – Trend, Seasonal, and Random Variations

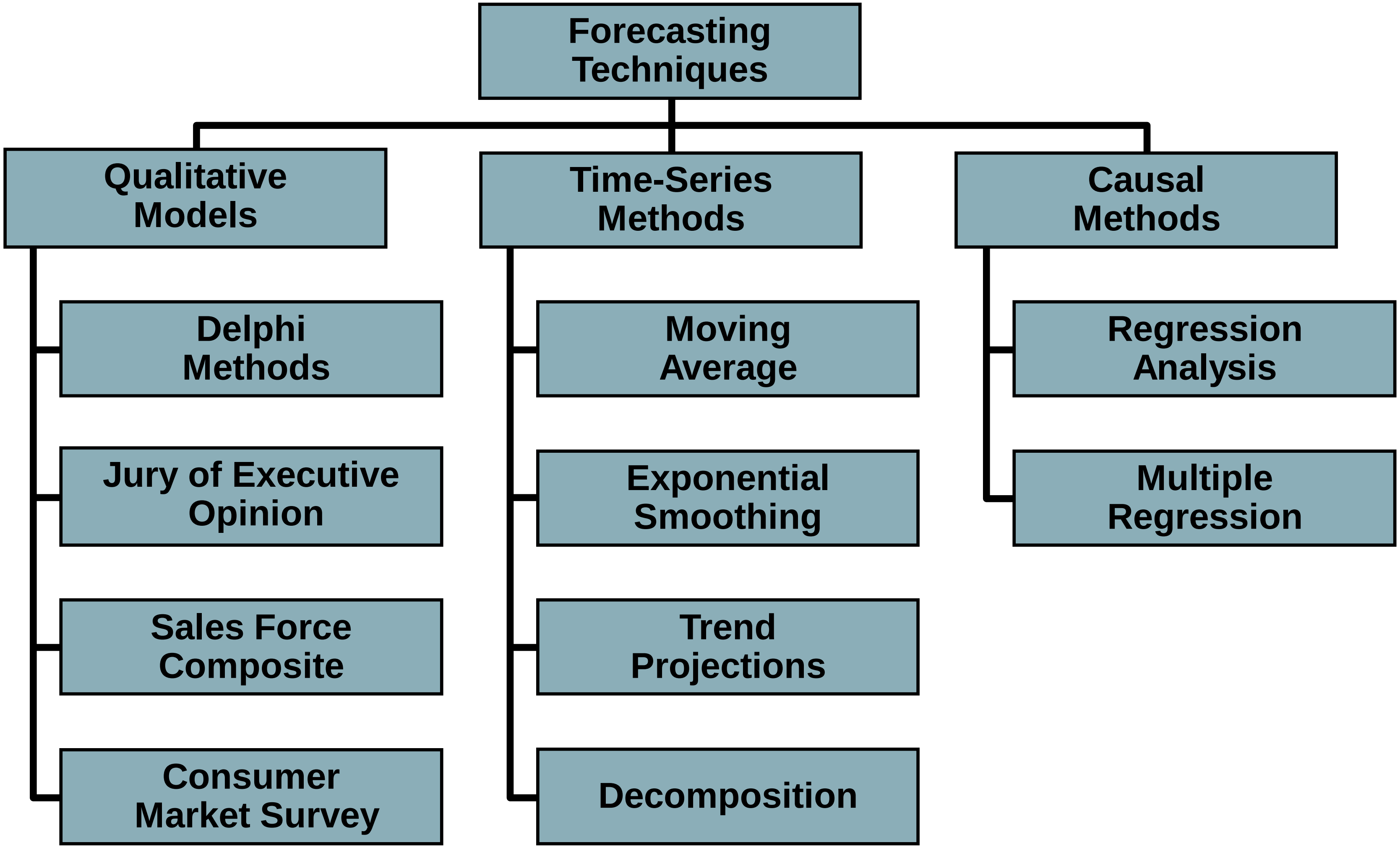
5.9 Monitoring and Controlling Forecasts

Introduction

- Main purpose of forecasting
  - Reduce uncertainty and make better estimates of what will happen in the future
- Subjective methods
  - Seat-of-the pants methods, intuition, experience
- More formal quantitative and qualitative techniques

Forecasting Models

FIGURE 5.1



Qualitative Models

- Incorporate judgmental or subjective factors
  - Useful when subjective factors are important or accurate quantitative data is difficult to obtain
- Common qualitative techniques
  1. *Delphi method*
  2. *Jury of executive opinion*
  3. *Sales force composite*
  4. *Consumer market surveys*

Qualitative Models

- *Delphi Method*
  - Iterative group process
  - Respondents provide input to decision makers
  - Repeated until consensus is reached
- *Jury of Executive Opinion*
  - Collects opinions of a small group of high-level managers
  - May use statistical models for analysis

Qualitative Models

- *Sales Force Composite*
  - Allows individual salespersons estimates
  - Reviewed for reasonableness
  - Data is compiled at a district or national level
- *Consumer Market Survey*
  - Information on purchasing plans solicited from customers or potential customers
  - Used in forecasting, product design, new product planning

Time-Series Models

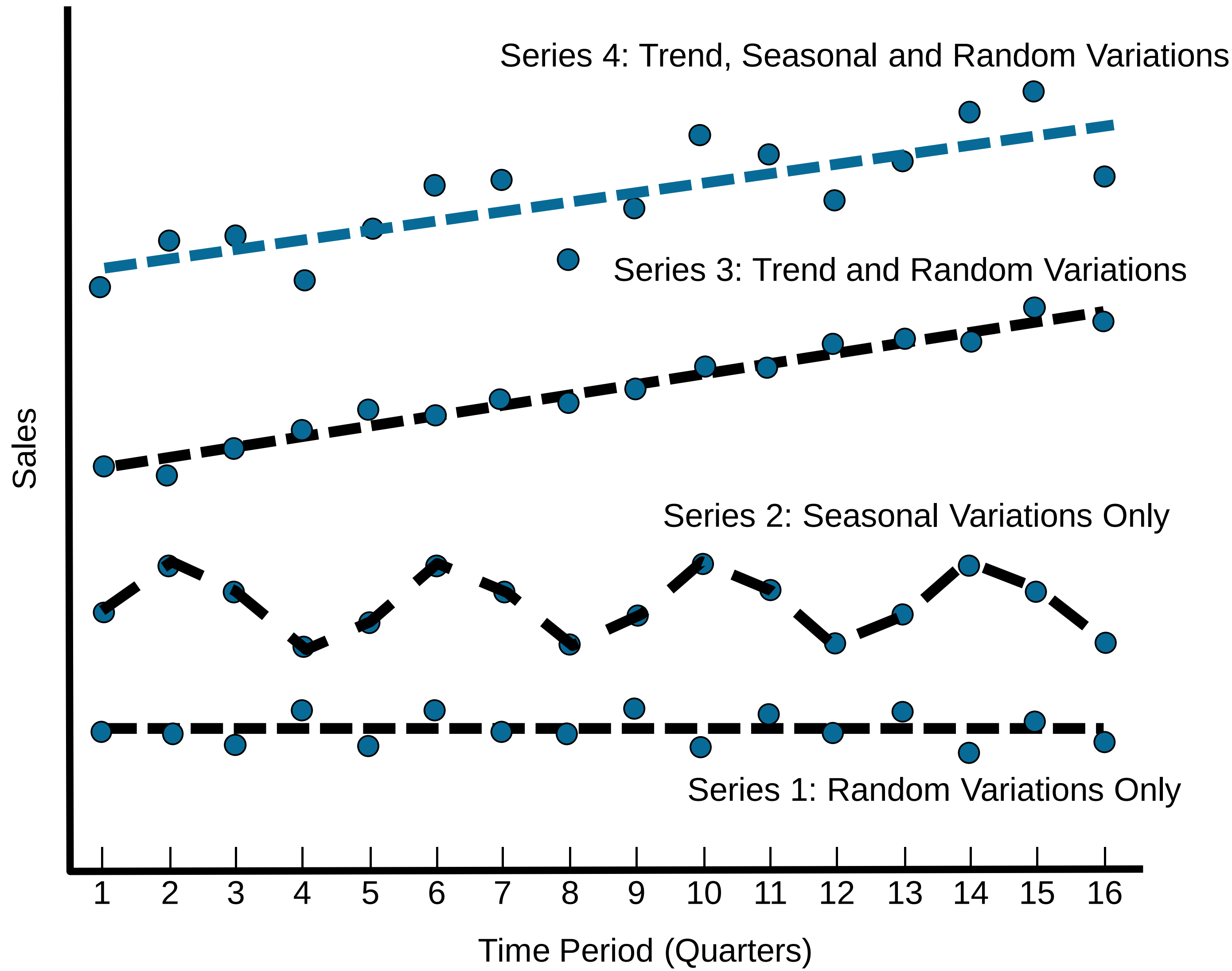
- Predict the future based on the past
- Uses only historical data on one variable
- Extrapolations of past values of a series
- Ignores factors such as
  - Economy
  - Competition
  - Selling price

Components of a Time Series

- Sequence of values recorded at successive intervals of time
- Four possible components
  - Trend (T)
  - Seasonal (S)
  - Cyclical (C)
  - Random (R)

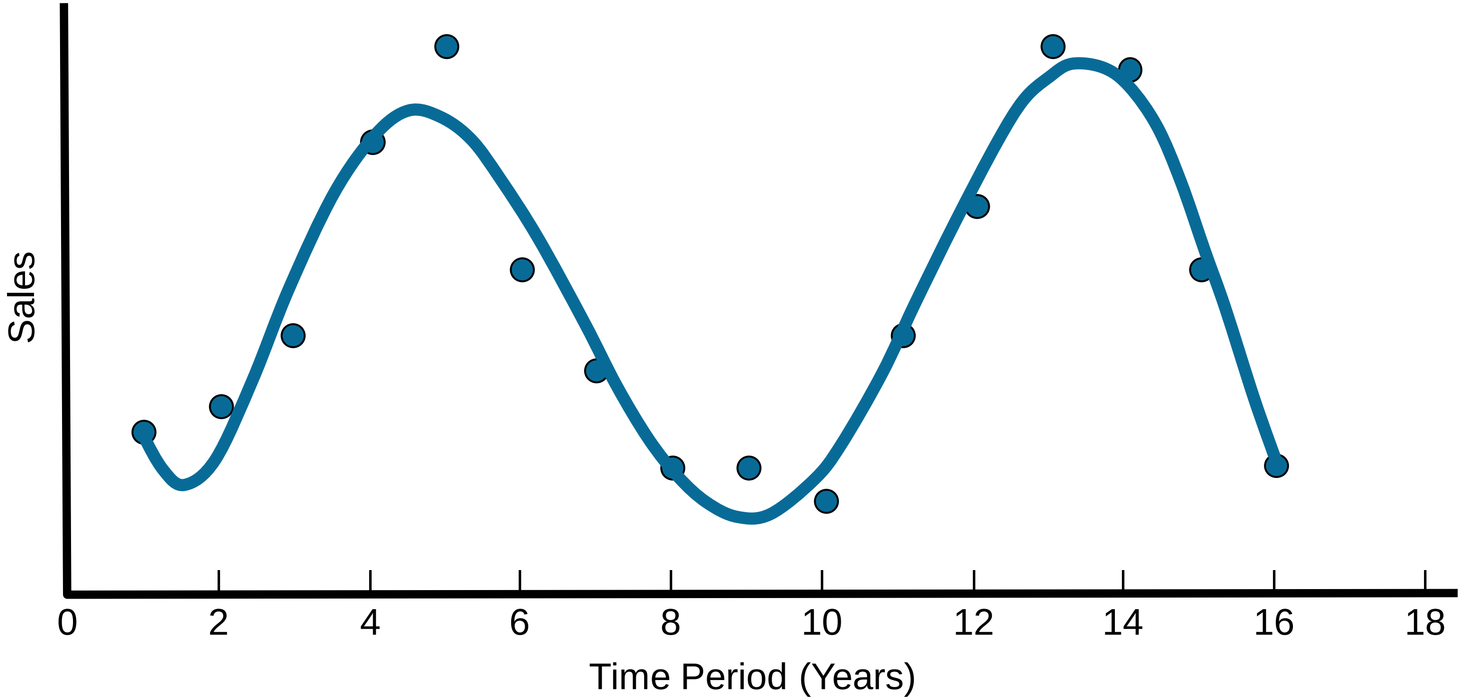
Components of a Time Series

FIGURE 5.2 – Scatter Diagram for Four Time Series of Quarterly Data



Components of a Time Series

FIGURE 5.3 – Scatter Diagram of Times Series with Cyclical and Random Components



Time-Series Models

- Two basic forms
  - Multiplicative

$$\text{Demand} = T \times S \times C \times R$$

  - Additive

$$\text{Demand} = T + S + C + R$$

  - Combinations are possible

Measures of Forecast Accuracy

- Compare forecasted values with actual values
  - See how well one model works
  - To compare models

Forecast error = Actual value – Forecast value

- Measure of accuracy
  - **Mean absolute deviation (MAD)**:

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n}$$

Measures of Forecast Accuracy

TABLE 5.1 – Computing the Mean Absolute Deviation (MAD)

YEAR	ACTUAL SALES OF WIRELESS SPEAKERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST)
1	110	—	
2	100	110	
3	120	100	
4	140	120	
5	170	140	
6	150	170	
7	160	150	
8	190	160	
9	200	190	
10	190	200	
11	—	190	

Measures of Forecast Accuracy

TABLE 5.1 – Computing the Mean Absolute Deviation (MAD)

YEAR	ACTUAL SALES OF WIRELESS SPEAKERS	FORECAST SALES
1	110	—
2	100	110
3	120	100
4	140	120
5	170	140
6	150	170
7	160	150
8	190	160
9	200	190
10	190	200
11	—	190

- Forecast based on naïve model
- No attempt to adjust for time series components

$$MAD = \frac{\sum |\text{forecast error}|}{n} = \frac{160}{9} = 17.8$$

Accuracy

YEAR	ACTUAL SALES OF WIRELESS SPEAKERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST)
1	110	—	—
2	100	110	|100 – 110| = 10
3	120	100	|120 – 110| = 20
4	140	120	|140 – 120| = 20
5	170	140	|170 – 140| = 30
6	150	170	|150 – 170| = 20
7	160	150	|160 – 150| = 10
8	190	160	|190 – 160| = 30
9	200	190	|200 – 190| = 10
10	190	200	|190 – 200| = 10
11	—	190	—
			Sum of |errors| = 160
			MAD = 160/9 = 17.8

Measures of Forecast Accuracy

- Other common measures
  - **Mean squared error (MSE)**

$$\text{MSE} = \frac{\sum (\text{error})^2}{n}$$

  - **Mean absolute percent error (MAPE)**

$$\text{MAPE} = \frac{\sum \left|\frac{\text{error}}{\text{actual}}\right|}{n} 100\%$$

  - **Bias** is the average error

Forecasting Random Variations

- No other components are present
- Averaging techniques smooth out forecasts
  - Moving averages
  - Weighted moving averages
  - Exponential smoothing

Moving Averages

- Used when demand is relatively steady over time
  - The next forecast is the average of the most recent n data values from the time series
  - Smooths out short-term irregularities in the data series

$$\text{Moving average forecast} = \frac{\text{Sum of demands in previous } n \text{ periods}}{n}$$

Moving Averages

- Mathematically

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \ldots + Y_{t-n+1}}{n}$$

where

F_{t+1} = forecast for time period $t + 1$

Y_t = actual value in time period t

n = number of periods to average

Wallace Garden Supply

- Wallace Garden Supply wants to forecast demand for its Storage Shed
  - Collected data for the past year
  - Use a three-month moving average ($n = 3$)

Wallace Garden Supply

TABLE 5.2

MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	(10 + 12 + 13)/3 = 11.67
May	19	(12 + 13 + 16)/3 = 13.67
June	23	(13 + 16 + 19)/3 = 16.00
July	26	(16 + 19 + 23)/3 = 19.33
August	30	(19 + 23 + 26)/3 = 22.67
September	28	(23 + 26 + 30)/3 = 26.33
October	18	(26 + 30 + 28)/3 = 28.00
November	16	(30 + 28 + 18)/3 = 25.33
December	14	(28 + 18 + 16)/3 = 20.67
January	—	(18 + 16 + 14)/3 = 16.00

Weighted Moving Averages

- *Weighted moving averages* use weights to put more emphasis on previous periods
  - Often used when a trend or other pattern is emerging

$$F_{t+1} = \frac{\sum (\text{Weight in period } i)(\text{Actual value in period})}{\sum (\text{Weights})}$$

  - Mathematically

$$F_{t+1} = \frac{w_1 Y_t + w_2 Y_{t-1} + \ldots + w_n Y_{t-n+1}}{w_1 + w_2 + \ldots + w_n}$$

where

w_i = weight for the i^{th} observation

Wallace Garden Supply

- Use a 3-month weighted moving average model to forecast demand
  - Weighting scheme

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
3 x Sales last month + 2 x Sales two months ago + 1 X Sales three months ago	
6	Sum of the weights

Wallace Garden Supply

TABLE 5.3

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	[(3 X 13) + (2 X 12) + (10)]/6 = 12.17
May	19	[(3 X 16) + (2 X 13) + (12)]/6 = 14.33
June	23	[(3 X 19) + (2 X 16) + (13)]/6 = 17.00
July	26	[(3 X 23) + (2 X 19) + (16)]/6 = 20.50
August	30	[(3 X 26) + (2 X 23) + (19)]/6 = 23.83
September	28	[(3 X 30) + (2 X 26) + (23)]/6 = 27.50
October	18	[(3 X 28) + (2 X 30) + (26)]/6 = 28.33
November	16	[(3 X 18) + (2 X 28) + (30)]/6 = 23.33
December	14	[(3 X 16) + (2 X 18) + (28)]/6 = 18.67
January	—	[(3 X 14) + (2 X 16) + (18)]/6 = 15.33

Exponential Smoothing

- **Exponential smoothing**
  - A type of moving average
  - Easy to use
  - Requires little record keeping of data

New forecast = Last period's forecast
+ α(Last period's actual demand
– Last period's forecast)

α is a weight (or **smoothing constant**) with a value $0 \leq \alpha \leq 1$

Exponential Smoothing

- Mathematically

$$F_{t+1} = F_t + a(Y_t - F_t)$$

where

F_{t+1} = new forecast (for time period t + 1)

Y_t = pervious forecast (for time period t)

α = smoothing constant ($0 \leq \alpha \leq 1$)

Y_t = pervious period’s actual demand

The idea is simple – the new estimate is the old estimate plus some fraction of the error in the last period

Exponential Smoothing Example

- In January, February's demand for a certain car model was predicted to be 142
- Actual February demand was 153 autos
- Using a smoothing constant of $\alpha = 0.20$, what is the forecast for March?

New forecast (for March demand) = 142 + 0.2(153 – 142)
= 144.2 or 144 autos

- If actual March demand = 136

New forecast (for April demand) = 144.2 + 0.2(136 – 144.2)
= 142.6 or 143 autos

Selecting the Smoothing Constant

- Selecting the appropriate value for α is key to obtaining a good forecast
- The objective is always to generate an accurate forecast
- The general approach is to develop trial forecasts with different values of α and select the α that results in the lowest MAD

Port of Baltimore Example

TABLE 5.4 – Exponential Smoothing Forecast for $\alpha = 0.1$ and $\alpha = 0.5$

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST USING $\alpha = 0.10$	FORECAST USING $\alpha = 0.50$
1	180	175	175
2	168	175.5 = 175.00 + 0.10(180 – 175)	177.5
3	159	174.75 = 175.50 + 0.10(168 – 175.50)	172.75
4	175	173.18 = 174.75 + 0.10(159 – 174.75)	165.88
5	190	173.36 = 173.18 + 0.10(175 – 173.18)	170.44
6	205	175.02 = 173.36 + 0.10(190 – 173.36)	180.22
7	180	178.02 = 175.02 + 0.10(205 – 175.02)	192.61
8	182	178.22 = 178.02 + 0.10(180 – 178.02)	186.30
9	?	178.60 = 178.22 + 0.10(182 – 178.22)	184.15

Port of Baltimore Example

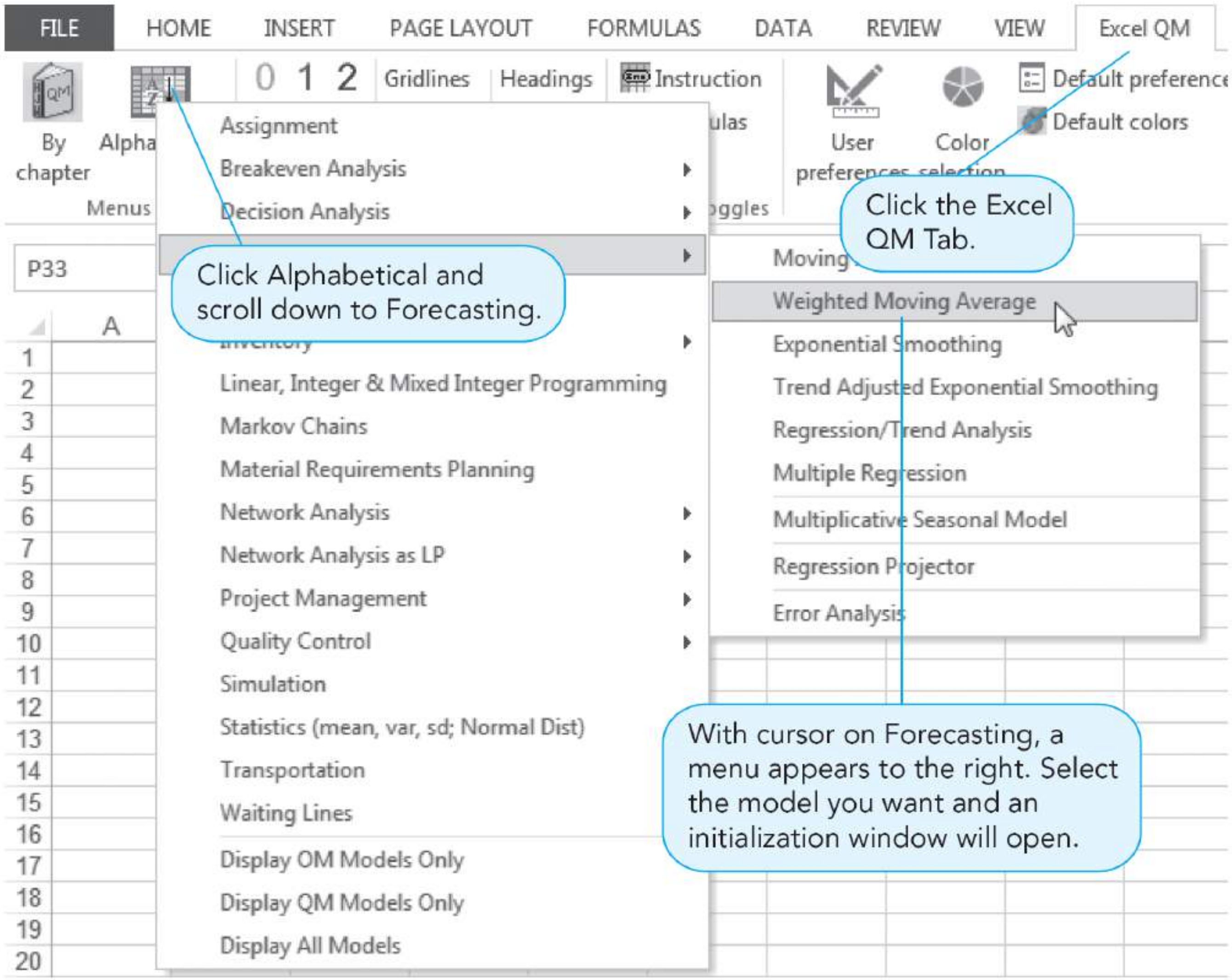
TABLE 5.5 – Absolute Deviations and MADs

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = 0.10$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.10$	FORECAST WITH $\alpha = 0.50$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.50$
1	180	175	5	175	5
2	168	175.5	7.5	177.5	9.5
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.3
Sum of absolute deviations			82.45		98.63
		$\text{MAD} = \dfrac{\Sigma\lvert\text{deviations}\rvert}{n} =$	10.31	MAD =	12.33

Best choice

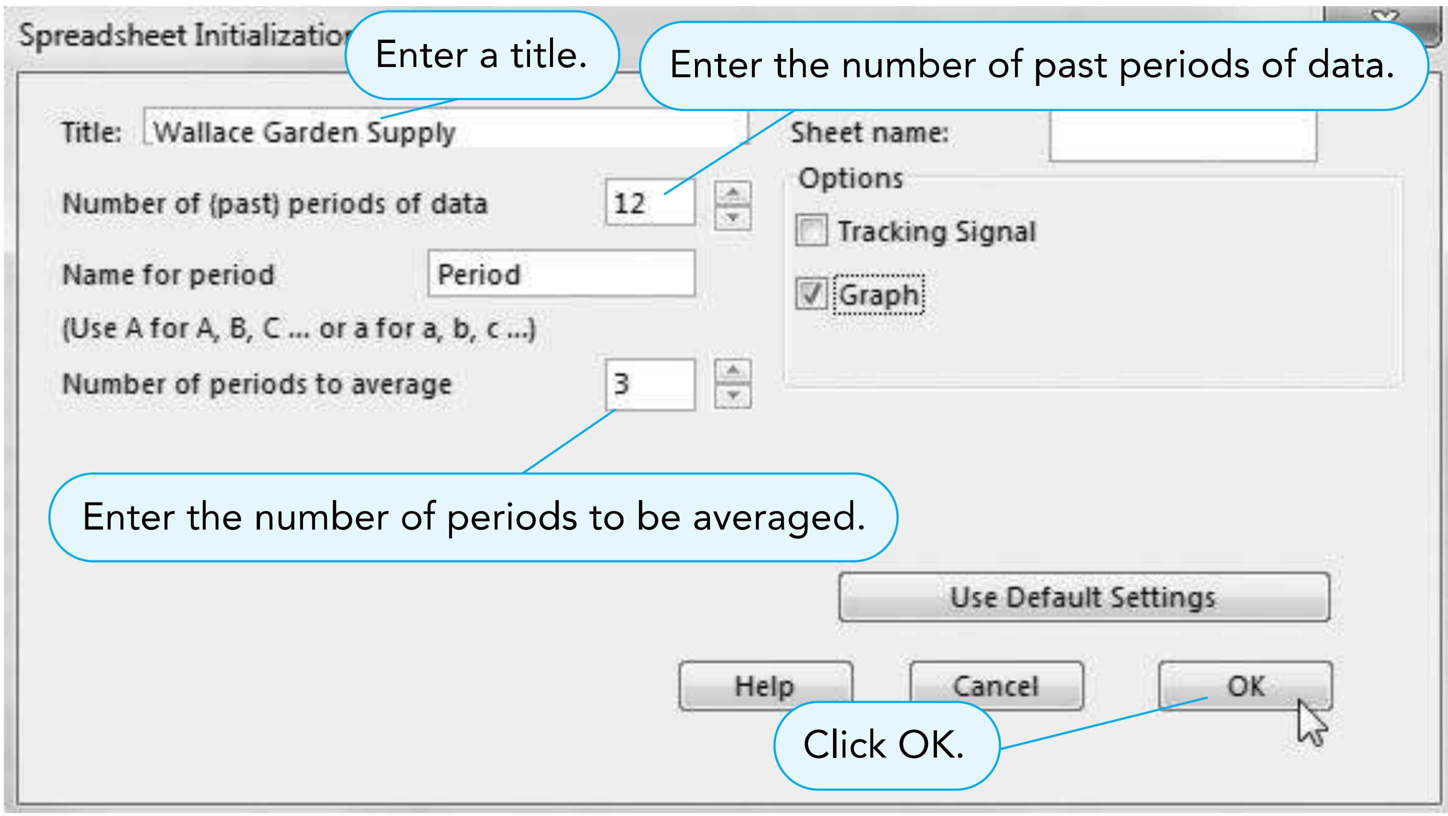
Using Software

PROGRAM 5.1A – Selecting the Forecasting Model



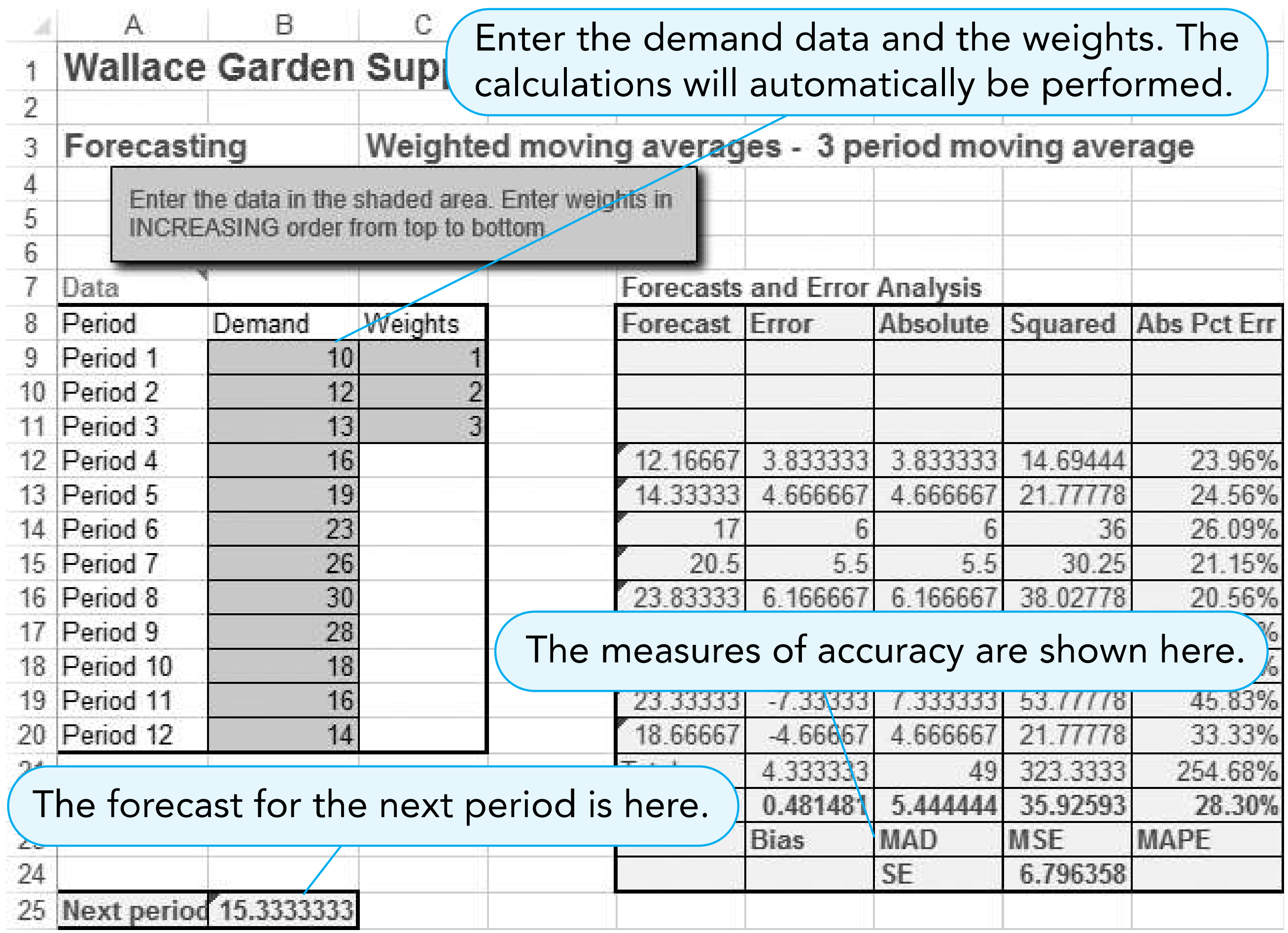
Using Software

PROGRAM 5.1B – Initializing Excel QM



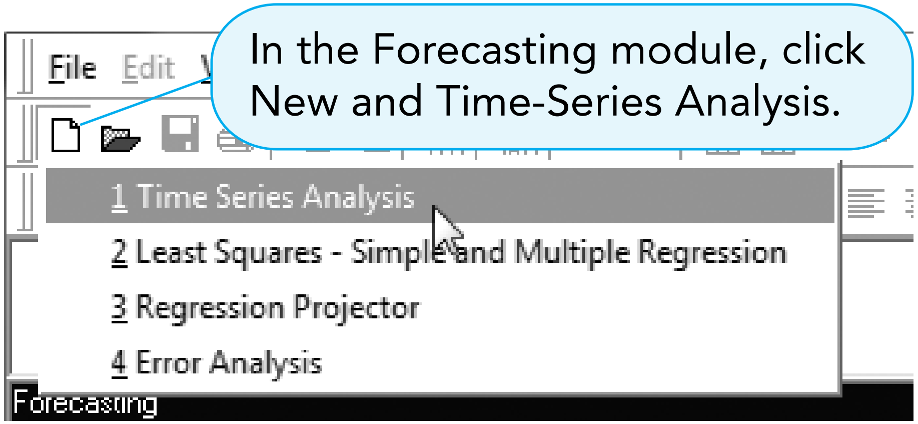
Using Software

PROGRAM 5.1C – Excel QM Output



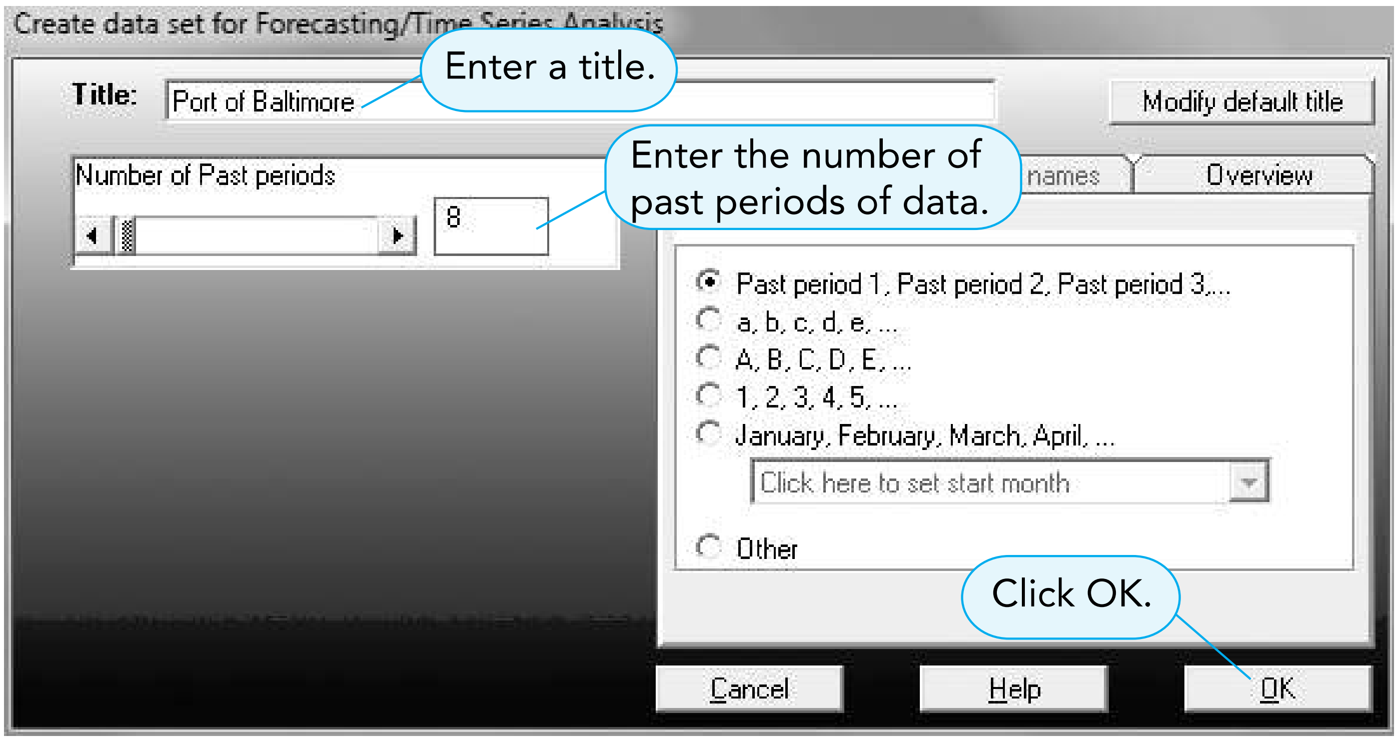
Using Software

PROGRAM 5.2A – Selecting Time-Series Analysis in QM for Windows



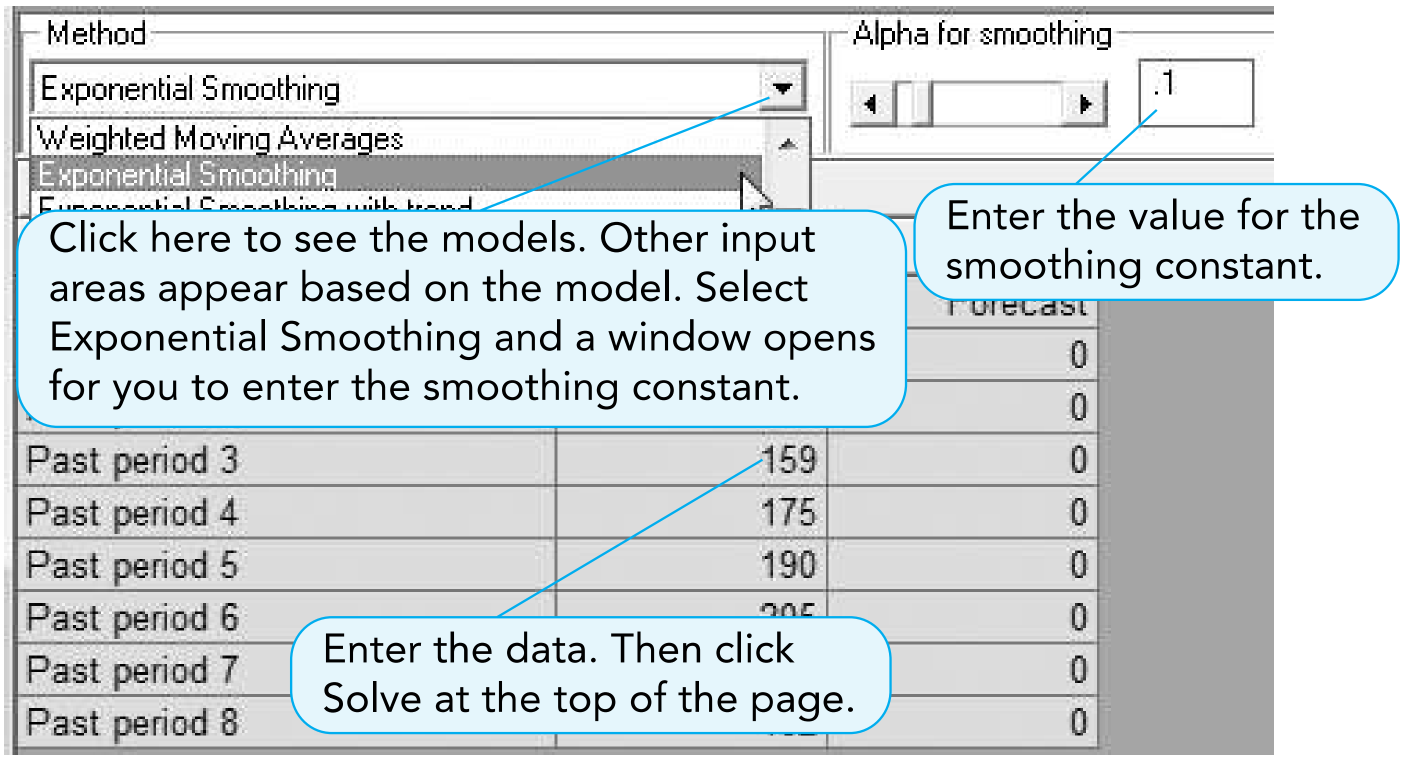
Using Software

PROGRAM 5.2B – Entering Data



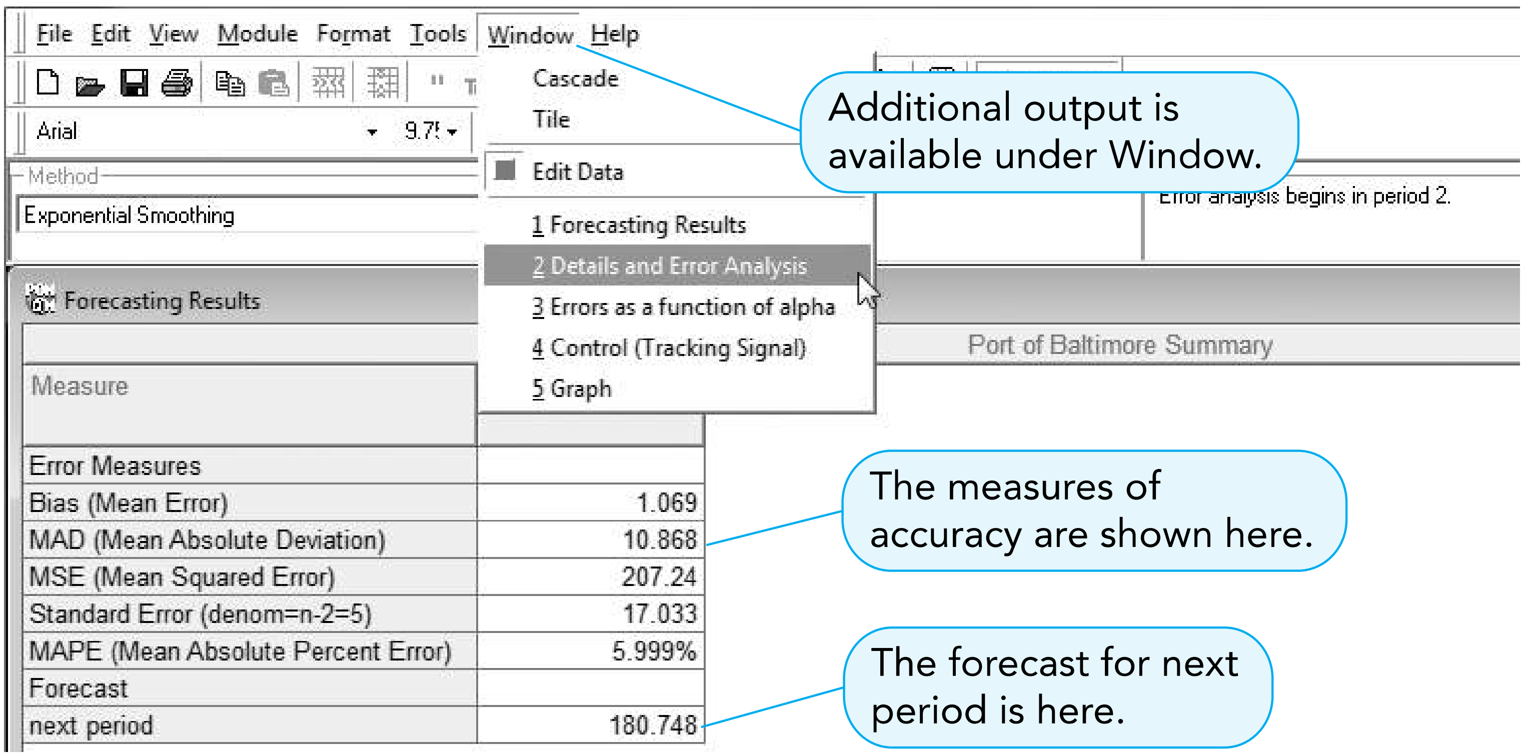
Using Software

PROGRAM 5.2C – Selecting the Model and Entering Data



Using Software

PROGRAM 5.2D – Output for Port of Baltimore Example



Forecasting – Trend and Random

- Exponential smoothing does not respond to trends
- A more complex model can be used
- The basic approach
  - Develop an exponential smoothing forecast
  - Adjust it for the trend

$$\text{Forecast including trend } (FIT_{t+1}) = \text{Smoothed forecast } (F_{t+1}) + \text{Smoothed Trend } (T_{t+1})$$

Exponential Smoothing with Trend

- The equation for the trend correction uses a new smoothing constant β
- F_t and T_t must be given or estimated
- Three steps in developing FIT_t

Step 1: Compute smoothed forecast F_{t+1}

$$\text{Smoothed forecast} = \text{Previous forecast including trend} + \alpha(\text{Last error})$$

$$F_{t+1} = FIT_t + a(Y_t - FIT_t)$$

Exponential Smoothing with Trend

Step 2: Update the trend (T_{t+1}) using

Smoothed forecast = Previous forecast including trend + β(Error or excess in trend)

$$T_{t+1} = T_t + b(F_{t+1} - FIT_t)$$

Step 3: Calculate the trend-adjusted exponential smoothing forecast (FIT_{t+1}) using

Forecast including trend (FIT_{t+1}) = Smoothed forecast (F_{t+1}) + Smoothed trend (T_{t+1})

$$FIT_{t+1} = F_{t+1} + T_{t+1}$$

Selecting a Smoothing Constant

- A high value of β makes the forecast more responsive to changes in trend
- A low value of β gives less weight to the recent trend and tends to smooth out the trend
- Values are often selected using a trial-and-error approach based on the value of the MAD for different values of β

Midwestern Manufacturing

- Demand for electrical generators from 2007 – 2013
  - Midwest assumes F_1 is perfect, $T_1 = 0$, $\alpha = 0.3$, $\beta = 0.4$

$$FIT_1 = F_1 + T_1 = 74 + 0 = 74$$

TABLE 5.6 – Demand

YEAR	ELECTRICAL GENERATORS SOLD
2007	74
2008	79
2009	80
2010	90
2011	105
2012	142
2013	122

Midwestern Manufacturing

For 2008 (time period 2)

Step 1: Compute F_{t+1}

$$F_2 = FIT_1 + \alpha(Y_1 - FIT_1)$$
$$= 74 + 0.3(74 - 74) = 74$$

Step 2: Update the trend

$$T_2 = T_1 + \beta(F_2 - FIT_1)$$
$$= 0 + .4(74 - 74) = 0$$

Midwestern Manufacturing

Step 3: Calculate the trend-adjusted exponential smoothing forecast (F_{t+1}) using

$$FIT_2 = F_2 + T_2$$
$$= 74 + 0 = 74$$

Midwestern Manufacturing

For 2009 (time period 3)

Step 1: $F_3 = FIT_2 + \alpha(Y_2 - FIT_2)$

$= 74 + 0.3(79 - 74) = 75.5$

Step 2: $T_3 = T_2 + .4(F_3 - FIT_2)$

$= 0 + .4(75.5 - 74) = 0.6$

Step 3: $FIT_3 = F_3 + T_3$

$= 75.5 + 0.6 = 76.1$

Midwestern Manufacturing

TABLE 5.7 – Exponential Smoothing with Trend Forecasts

TIME (t)	DEMAND (Y_t)	$F_{t+1} = FIT_t + 0.3(Y_t - FIT_t)$	$T_{t+1} = T_t + 0.4(F_{t+1} - FIT_t)$	$FIT_{t+1} = F_{t+1} + T_{t+1}$
1	74	74	0	74
2	79	74 = 74 + 0.3(74 – 74)	0 = 0 + 0.4(74 – 74)	74 = 74 + 0
3	80	75.5 = 74 + 0.3(79 – 74)	0.6 = 0 + 0.4(75.5 – 74)	76.1 = 75.5 + 0.6
4	90	77.270 = 76.1 + 0.3(80 – 76.1)	1.068 = 0.6 + 0.4(77.27 – 76.1)	78.338 = 77.270 + 1.068
5	105	81.837 = 78.338 + 0.3(90 – 78.338)	2.468 = 1.068 + 0.4(81.837 – 78.338)	84.305 = 81.837 + 2.468
6	142	90.514 = 84.305 + 0.3(105 – 84.305)	4.952 = 2.468 + 0.4(90.514 – 84.305)	95.466 = 90.514 + 4.952
7	122	109.426 = 95.446 + 0.3(142 – 95.466)	10.536 = 4.952 + 0.4(109.426 – 95.466)	119.962 = 109.426 + 10.536
8		120.573 = 119.962 + 0.3(122 – 119.962)	10.780 = 10.536 + 0.4(120.573 – 119.962)	131.353 = 120.573 + 10.780

Midwestern Manufacturing

PROGRAM 5.3 – Output from Excel QM Trend-Adjusted Exponential Smoothing

	A	B	C	D	E	F	G	H	I	J
1	**Midwestern Manufacturing Company Example**									
2										
3	**Forecasting**		**Trend adjusted exponential smoothing**							
4–6	Enter alpha and beta (between 0 and 1), enter the past demands in the shaded column then enter a starting forecast. If the starting forecast is not in the first period then delete the error analysis for									
7	Alpha	0.3								
8	Beta	0.4								
9	Data			**Forecasts and Error Analysis**						
10	Period	Demand		Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t	Error	Absolute	Squared	Abs Pct Err
11	Period 1	74		74		74	0	0	0	00.00%
12	Period 2	79		74	0	74	5	5	25	06.33%
13	Period 3	80		75.5	0.6	76.1	3.9	3.9	15.21	04.88%
14	Period 4	90		77.27	1.068	78.338	11.662	11.662	136.0022	12.96%
15	Period 5	105		81.8366	2.46744	84.30404	20.69596	20.696	428.3228	19.71%
16	Period 6	142		90.512828	4.950955	95.4637832	46.53622	46.5362	2165.619	32.77%
17	Period 7	122		109.4246482	10.5353	119.959949	2.040051	2.04005	4.161806	0.016722
18		**Next period**		**120.5719646**	**10.78011**	**131.352072**				
19				Total			89.83423	89.8342	2774.316	78.32%
20				Average			**12.83346**	**12.8335**	**396.3309**	**11.19%**
21							**Bias**	**MAD**	**MSE**	**MAPE**
22								SE	**23.55554**	

The forecast for next period is here.

Trend Projections

- Fits a trend line to a series of historical data points
- Projected into the future for medium- to long-range forecasts
- Trend equations can be developed based on exponential or quadratic models
- Linear model developed using regression analysis is simplest

Trend Projections

- Mathematical formula

$$\hat{Y} = b_0 + b_1 X$$

where

$\hat{Y}$ = predicted value
b_0 = intercept
b_1 = slope of the line
X = time period (i.e., X = 1, 2, 3, …, n)

Midwestern Manufacturing

- Based on least squares regression, the forecast equation is

$$\hat{Y} = 56.71 + 10.54X$$

- Year 2014 is coded as $X = 8$

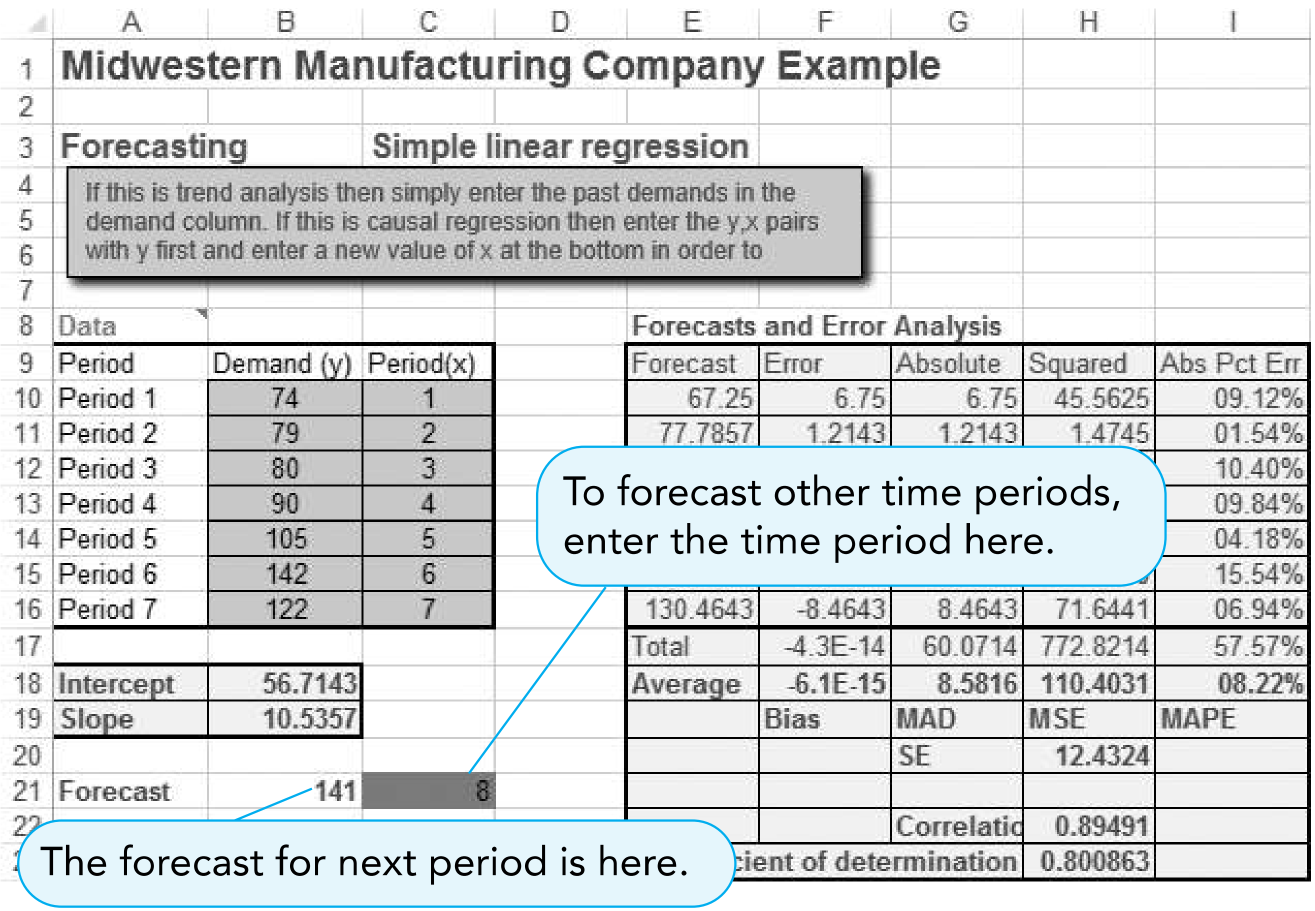
(sales in 2014) = 56.71 + 10.54(8)
= 141.03, or 141 generators

- For $X = 9$

(sales in 2015) = 56.71 + 10.54(9)
= 151.57, or 152 generators

Midwestern Manufacturing

PROGRAM 5.4 – Output from Excel QM for Trend Line



	A	B	C	D	E	F	G	H	I
1	Midwestern Manufacturing Company Example								
2									
3	Forecasting		Simple linear regression						
4	If this is trend analysis then simply enter the past demands in the								
5	demand column. If this is causal regression then enter the y,x pairs								
6	with y first and enter a new value of x at the bottom in order to								
7									
8	Data				Forecasts and Error Analysis				
9	Period	Demand (y)	Period(x)		Forecast	Error	Absolute	Squared	Abs Pct Err
10	Period 1	74	1		67.25	6.75	6.75	45.5625	09.12%
11	Period 2	79	2		77.7857	1.2143	1.2143	1.4745	01.54%
12	Period 3	80	3						10.40%
13	Period 4	90	4						09.84%
14	Period 5	105	5						04.18%
15	Period 6	142	6						15.54%
16	Period 7	122	7		130.4643	-8.4643	8.4643	71.6441	06.94%
17					Total	-4.3E-14	60.0714	772.8214	57.57%
18	Intercept	56.7143			Average	-6.1E-15	8.5816	110.4031	08.22%
19	Slope	10.5357				Bias	MAD	MSE	MAPE
20							SE	12.4324	
21	Forecast	141	8						
22							Correlatio	0.89491	
23						cient of determination		0.800863	

Midwestern Manufacturing

PROGRAM 5.5 – Output from QM for Trend Line

Forecasting Results

Midwestern Manufacturing Company Summary

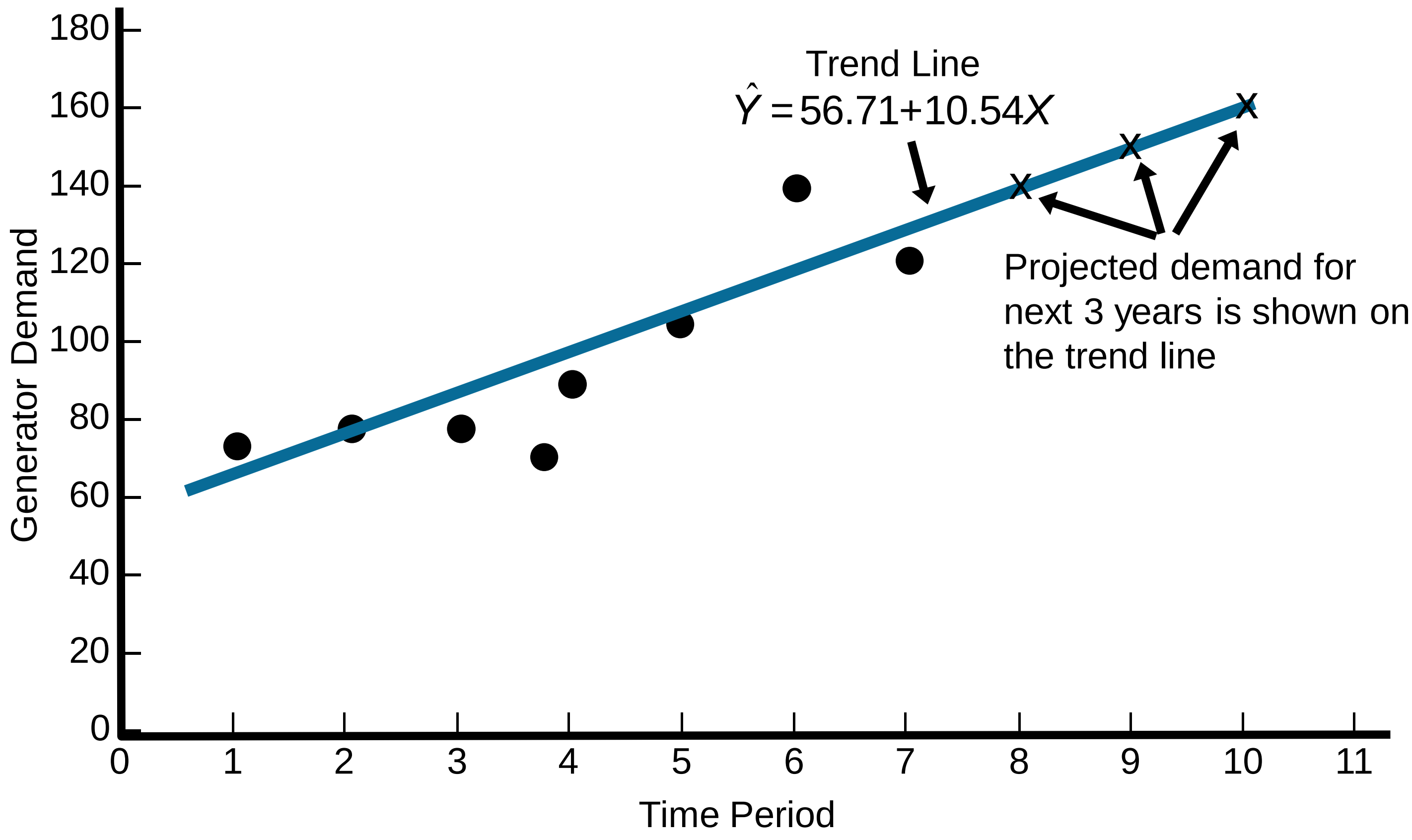
Measure		Future Period	Forecast
Error Measures		8	141
Bias (Mean Error)	0	9	151.536
MAD (Mean Absolute Deviation)	8.582	10	162.071
MSE (Mean Squared Error)	110.403	11	172.607
Standard Error (denom=n-2=5)	12.432	12	183.143
MAPE (Mean Absolute Percent	8.224%	13	193.679
Regression line		14	204.214
Demand(y) = 56.714		15	214.75
+ 10.536 * Time(x)		16	225.286
Statistics			235.822
Correlation coefficient			246.357
Coefficient of determination (r^2)			256.893
		20	267.429
		21	277.964

Forecasts for future time periods are shown here.

The trend line is shown over two lines.

Midwestern Manufacturing

FIGURE 5.4 – Generator Demand Based on Trend Line



Seasonal Variations

- Recurring variations over time may indicate the need for *seasonal* adjustments in the trend line
- A **seasonal index** indicates how a particular season compares with an average season
  - An index of 1 indicates an average season
  - An index > 1 indicates the season is higher than average
  - An index < 1 indicates a season lower than average

Seasonal Indices

- **Deseasonalized data** is created by dividing each observation by the appropriate seasonal index
- Once deseasonalized forecasts have been developed, values are multiplied by the seasonal indices
- Computed in two ways
  - Overall average
  - Centered-moving-average approach

Seasonal Indices with No Trend

- Divide average value for each season by the average of all data
  - Telephone answering machines at Eichler Supplies
  - Sales data for the past two years for one model
  - Create a forecast that includes seasonality

Seasonal Indices with No Trend

TABLE 5.8 – Answering Machine Sales and Seasonal Indices

MONTH	SALES DEMAND YEAR 1	SALES DEMAND YEAR 2	AVERAGE 2- YEAR DEMAND	MONTHLY DEMAND	AVERAGE SEASONAL INDEX
January	80	100	90	94	0.957
February	85	75	80	94	0.851
March	80	90	85	94	0.904
April	110	90	100	94	1.064
May	115	131	123	94	1.309
June	120	110	115	94	1.223
July	100	110	105	94	1.117
August	110	90	100	94	1.064
September	85	95	90	94	0.957
October	75	85	80	94	0.851
November	85	75	80	94	0.851
December	80	80	80	94	0.851
		Total average demand =	1,128		

$$\text{Average monthly demand} = \frac{1{,}128}{12 \text{ months}} = 94$$

$$\text{Seasonal index} = \frac{\text{Average 2-year demand}}{\text{Average monthly demand}}$$

Seasonal Indices with No Trend

- Calculations for the seasonal indices

Jan. $\frac{1,200}{12} \times 0.957 = 96$

Feb. $\frac{1,200}{12} \times 0.851 = 85$

Mar. $\frac{1,200}{12} \times 0.904 = 90$

Apr. $\frac{1,200}{12} \times 1.064 = 106$

May $\frac{1,200}{12} \times 1.309 = 131$

June $\frac{1,200}{12} \times 1.223 = 122$

July $\frac{1,200}{12} \times 1.117 = 112$

Aug. $\frac{1,200}{12} \times 1.064 = 106$

Sept. $\frac{1,200}{12} \times 0.957 = 96$

Oct. $\frac{1,200}{12} \times 0.851 = 85$

Nov. $\frac{1,200}{12} \times 0.851 = 85$

Dec. $\frac{1,200}{12} \times 0.851 = 85$

Seasonal Indices with Trend

- Changes could be due to trend, seasonal, or random
- **Centered moving average** (CMA) approach prevents trend being interpreted as seasonal
- Turner Industries sales contain both trend and seasonal components

Seasonal Indices with Trend

- Steps in CMA
  1. Compute the CMA for each observation (where possible)
  2. Compute the seasonal ratio = Observation/CMA for that observation
  3. Average seasonal ratios to get seasonal indices
  4. If seasonal indices do not add to the number of seasons, multiply each index by (Number of seasons)/(Sum of indices)

Turner Industries

TABLE 5.9 – Quarterly Sales Data

QUARTER	YEAR 1	YEAR 2	YEAR 3	AVERAGE
1	108	116	123	115.67
2	125	134	142	133.67
3	150	159	168	159.00
4	141	152	165	152.67
Average	131.00	140.25	149.50	140.25

Definite trend

Seasonal pattern

Turner Industries

- To calculate the CMA for quarter 3 of year 1, compare the actual sales with an average quarter centered on that time period
- Use 1.5 quarters before quarter 3 and 1.5 quarters after quarter 3
  - Take quarters 2, 3, and 4 and one half of quarters 1, year 1 and quarter 1, year 2

$$\text{CMA}(q3, y1) = \frac{0.5(108) + 125 + 150 + 141 + 0.5(116)}{4} = 132.0$$

Turner Industries

- Compare the actual sales in quarter 3 to the CMA to find the seasonal ratio

$$\text{Seasonal ratio} = \frac{\text{Sales in quarter 3}}{\text{CMA}} = \frac{150}{132.0} = 1.136$$

Turner Industries

TABLE 5.10 – Centered Moving Averages and Seasonal Ratios

YEAR	QUARTER	SALES	CMA	SEASONAL RATIO
1	1	108		
	2	125		
	3	150	132.000	1.136
	4	141	134.125	1.051
2	1	116	136.375	0.851
	2	134	138.875	0.965
	3	159	141.125	1.127
	4	152	143.000	1.063
3	1	123	145.125	0.848
	2	142	147.875	0.960
	3	168		
	4	165		

Turner Industries

- The two seasonal ratios for each quarter are averaged to get the seasonal index

Index for quarter 1 = I_1 = (0.851 + 0.848)/2 = 0.85

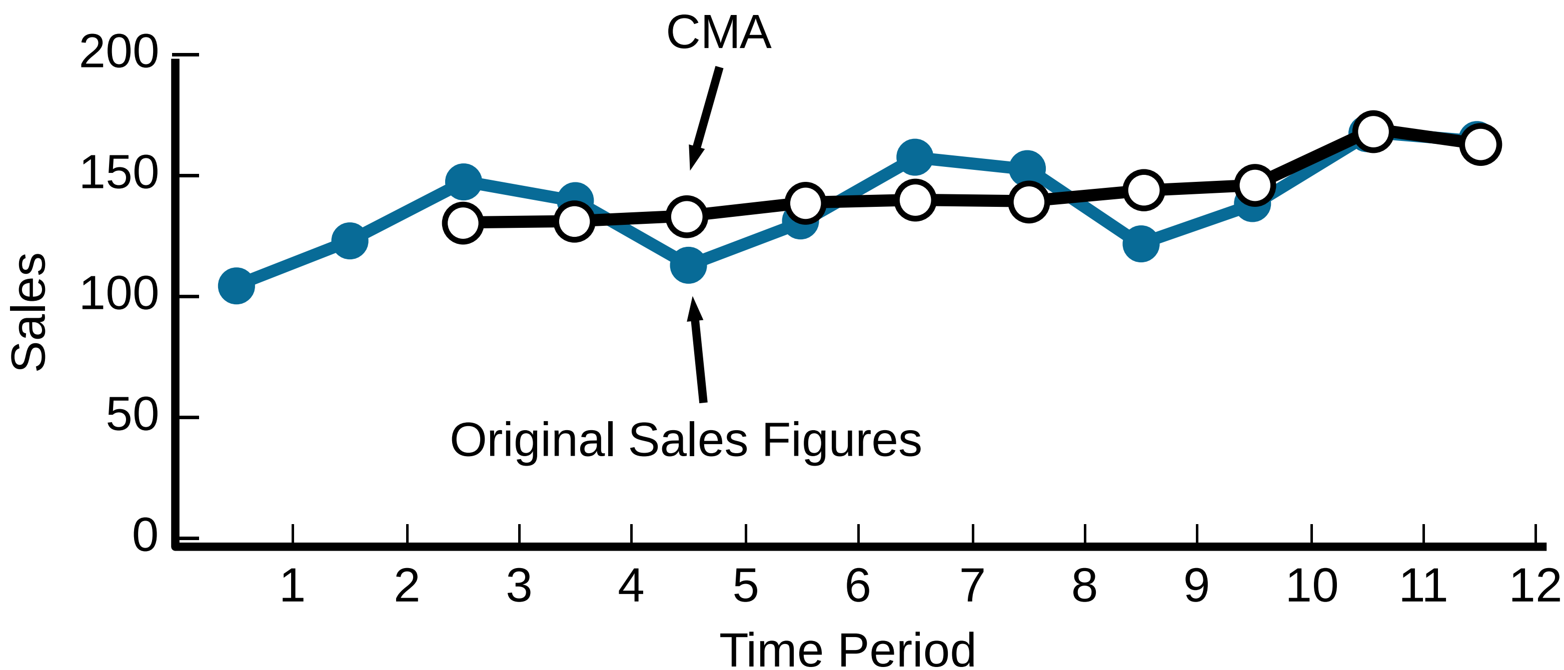
Index for quarter 2 = I_2 = (0.965 + 0.960)/2 = 0.96

Index for quarter 3 = I_3 = (1.136 + 1.127)/2 = 1.13

Index for quarter 4 = I_4 = (1.051 + 1.063)/2 = 1.06

Turner Industries

- Scatterplot of Turner Industries Sales Data and Centered Moving Average



Trend, Seasonal, and Random Variations

- **Decomposition** – isolating linear trend and seasonal factors to develop more accurate forecasts
- Five steps to decomposition
  - Compute seasonal indices using CMAs.
  - Deseasonalize the data by dividing each number by its seasonal index
  - Find the equation of a trend line using the deseasonalized data
  - Forecast for future periods using the trend line
  - Multiply the trend line forecast by the appropriate seasonal index

Deseasonalized Data

TABLE 5.11

SALES ($1,000,000s)	SEASONAL INDEX	DESEASONALIZED SALES ($1,000,000s)
108	0.85	127.059
125	0.96	130.208
150	1.13	132.743
141	1.06	133.019
116	0.85	136.471
134	0.96	139.583
159	1.13	140.708
152	1.06	143.396
123	0.85	144.706
142	0.96	147.917
168	1.13	148.673
165	1.06	155.660

Deseasonalized Data

- Find a trend line using the deseasonalized data where X = time

$$b_1 = 2.34 \quad b_0 = 124.78$$

$$\hat{Y} = 124.78 + 2.34X$$

- Develop a forecast for quarter 1, year 4 (X = 13) using this trend and multiply the forecast by the appropriate seasonal index

$$\hat{Y} = 124.78 + 2.34(13)$$
$$= 155.2 \text{ (before seasonality adjustment)}$$

Deseasonalized Data

- Find a trend line using the deseasonalized data wh

Including the seasonal index

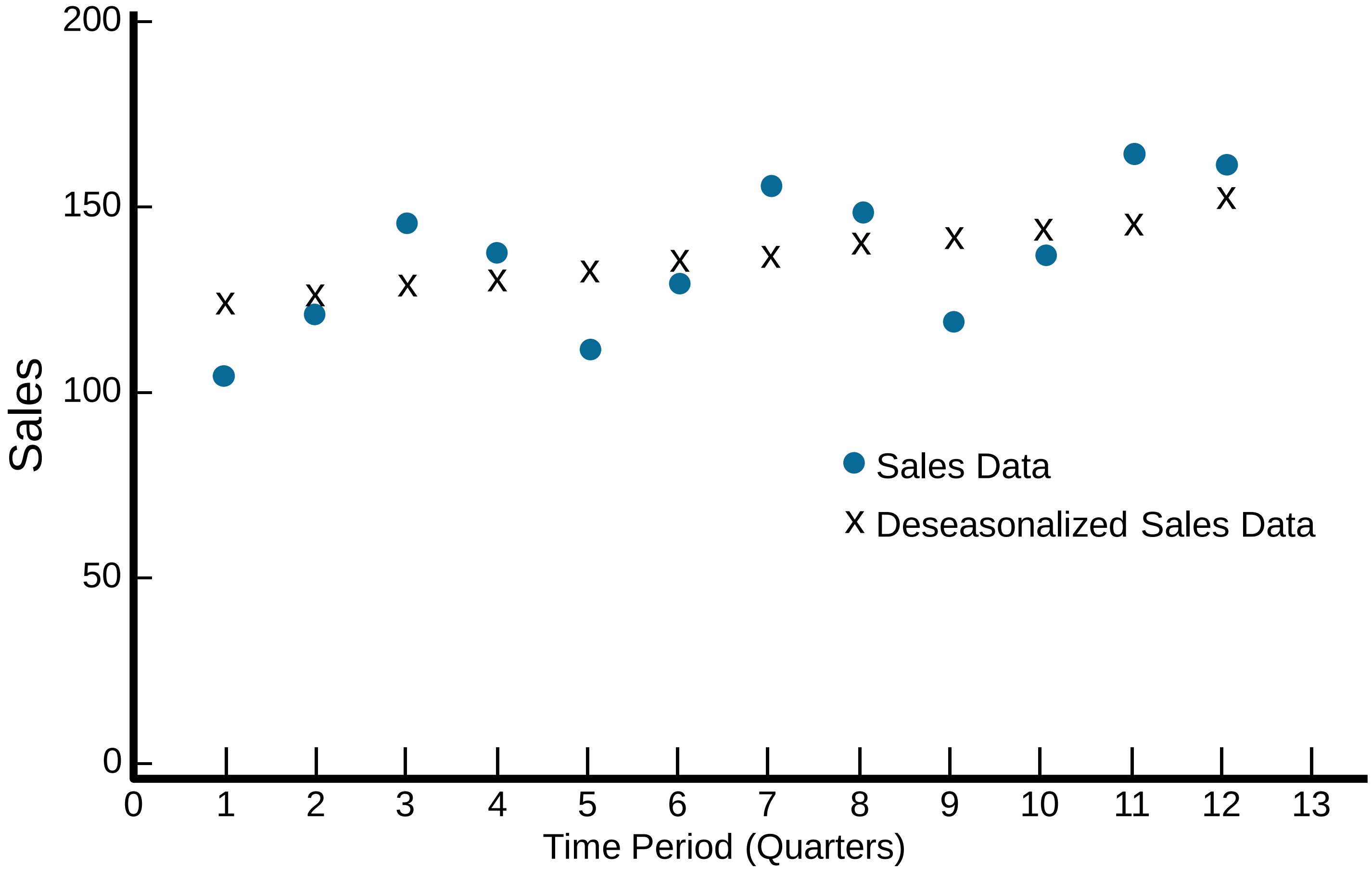
$$\hat{Y} \times I_1 = 155.2 \times 0.85 = 131.92$$

- Develop a forecast for quarter 1, year 4 (X = 13) using this trend and multiply the forecast by the appropriate seasonal index

$$\hat{Y} = 124.78 + 2.34(13)$$
$$= 155.2 \text{ (before seasonality)}$$

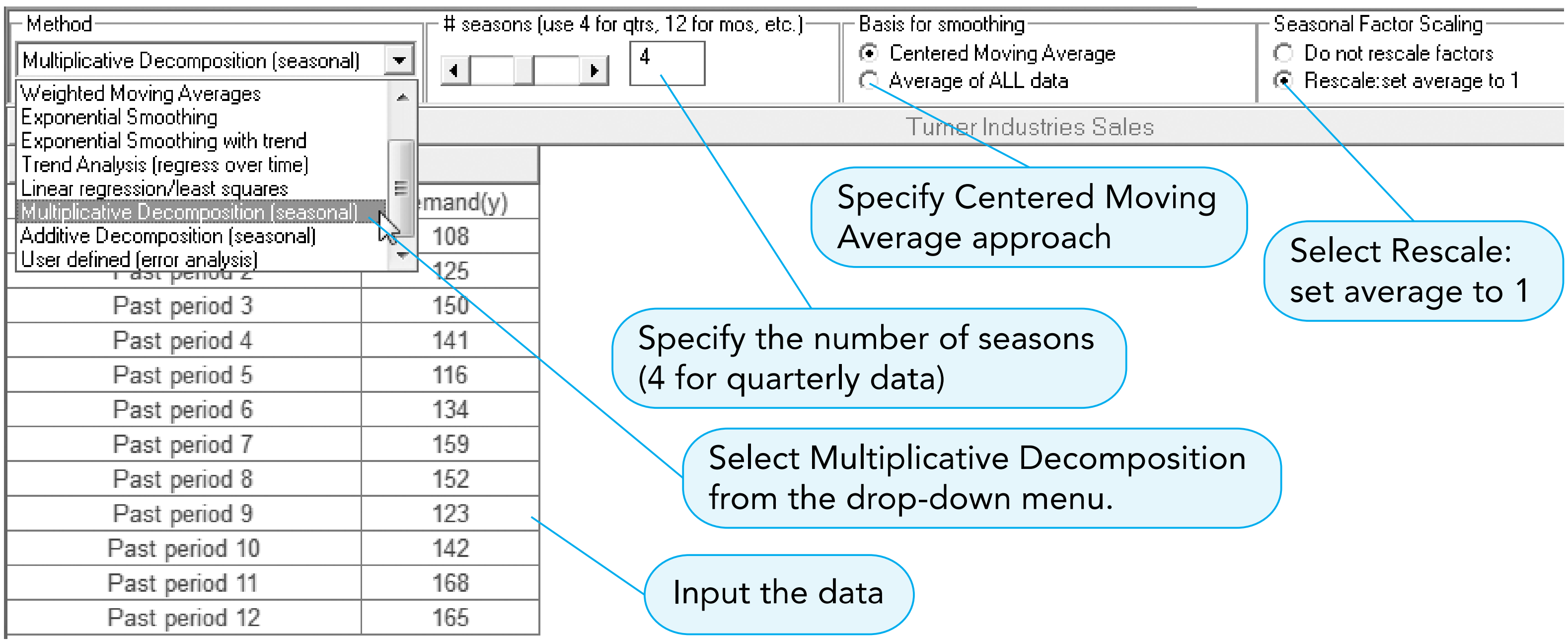
Deseasonalized Data

FIGURE 5.5



Using Software

PROGRAM 5.6A – QM for Windows Input



Using Software

PROGRAM 5.6B – QM for Windows Output

Method: Multiplicative Decomposition (seasonal)

seasons (use 4 for qtrs, 12 ...: 4

Forecasting Results

Measure			Unadjusted Forecast	Seasonal Factor	Adjusted Forecast
Error Measures		13	155.25	0.849	131.81
Bias (Mean Error)	0.001	14	157.594	0.963	151.687
MAD (Mean Absolute Deviation)	0.905	15	159.937	1.131	180.959
MSE (Mean Squared Error)	1.7	16	162.281	1.057	171.535
Standard Error (denom=n-2-4=6)	1.844	17	164.625	0.849	139.769
MAPE (Mean Absolute Percent Error)	0.595%	18	166.968	0.963	160.71
Regression line (unadjusted forecast)		19	169.312	1.131	191.566
Demand(y) = 124.784		20	171.655	1.057	181.444
+ 2.344 * time		21	173.999	0.849	147.728
Statistics		22	176.342	0.963	169.733
Correlation coeffici	8	23	178.686	1.131	202.172
Coefficient of detern	5	24	181.03	1.057	191.353
		25	183.373	0.849	155.686
		26	185.717	0.963	178.756

The final forecast is obtained by multiplying the trend (unadjusted) forecast by the seasonal indices (factors).

The seasonal indices (factors) are shown here.

The trend line is shown here over two lines.

Using Regression with Trend and Seasonal

- Multiple regression can be used to forecast both trend and seasonal components
  - One independent variable is time
  - Dummy independent variables are used to represent the seasons
  - An additive decomposition model

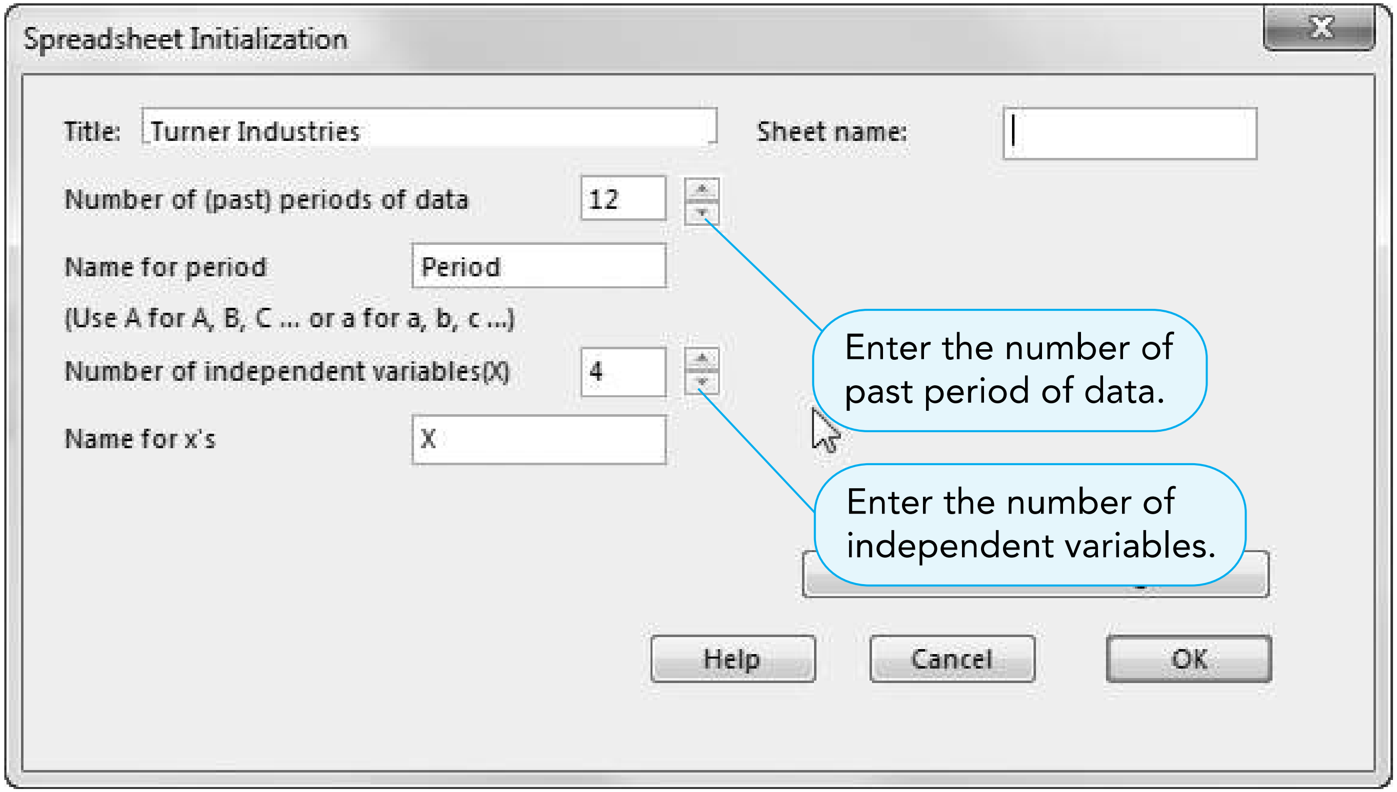
$$\hat{Y} = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$$

where

X_1 = time period
X_2 = 1 if quarter 2, 0 otherwise
X_3 = 1 if quarter 3, 0 otherwise
X_4 = 1 if quarter 4, 0 otherwise

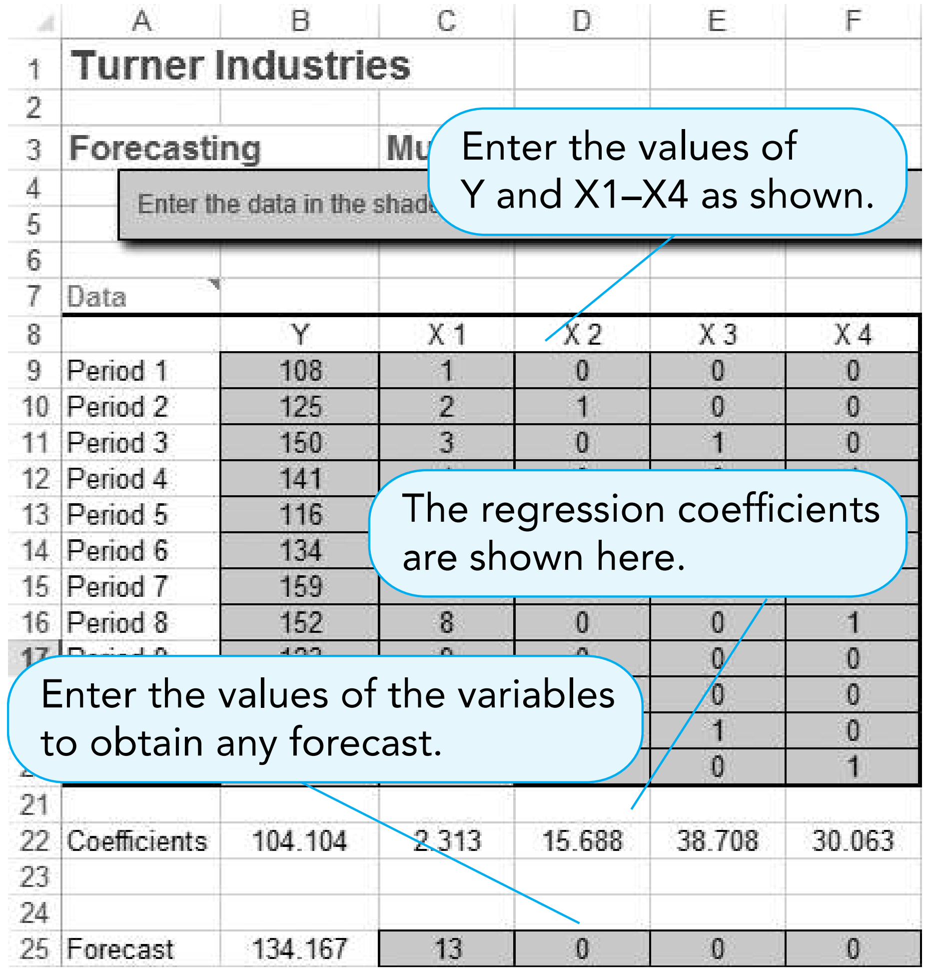
Using Regression with Trend and Seasonal

PROGRAM 5.7A – Excel QM Multiple Regression Initialization



Using Regression with Trend and Seasonal

PROGRAM 5.7B – Excel QM Multiple Regression Output



Using Regression with Trend and Seasonal

- Regression equation

$$\hat{Y} = 104.1 + 2.3X_1 + 15.7X_2 + 38.7X_3 + 30.1X_4$$

- Forecasts for first two quarters next year

$$\hat{Y} = 104.1 + 2.3(13) + 15.7(0) + 38.7(0) + 30.1(0) = 134$$

$$\hat{Y} = 104.1 + 2.3(14) + 15.7(1) + 38.7(0) + 30.1(0) = 152$$

Using Regression with Trend and Seasonal

- Regressi

$$\hat{Y} = 104$$

- Different from the results using the multiplicative decomposition method
- Use MAD or MSE to determine the best model

- Forecasts for first two quarters next year

$$\hat{Y} = 104.1 + 2.3(13) + 15.7(0) + 38.7(0) + 30.1(0) = 134$$

$$\hat{Y} = 104.1 + 2.3(14) + 15.7(1) + 38.7(0) + 30.1(0) = 152$$

Monitoring and Controlling Forecasts

- **Tracking signal** measures how well a forecast predicts actual values
  - **Running sum of forecast errors** (**RSFE**) divided by the MAD

$$\text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}} = \frac{\sum (\text{forecast error})}{\text{MAD}}$$

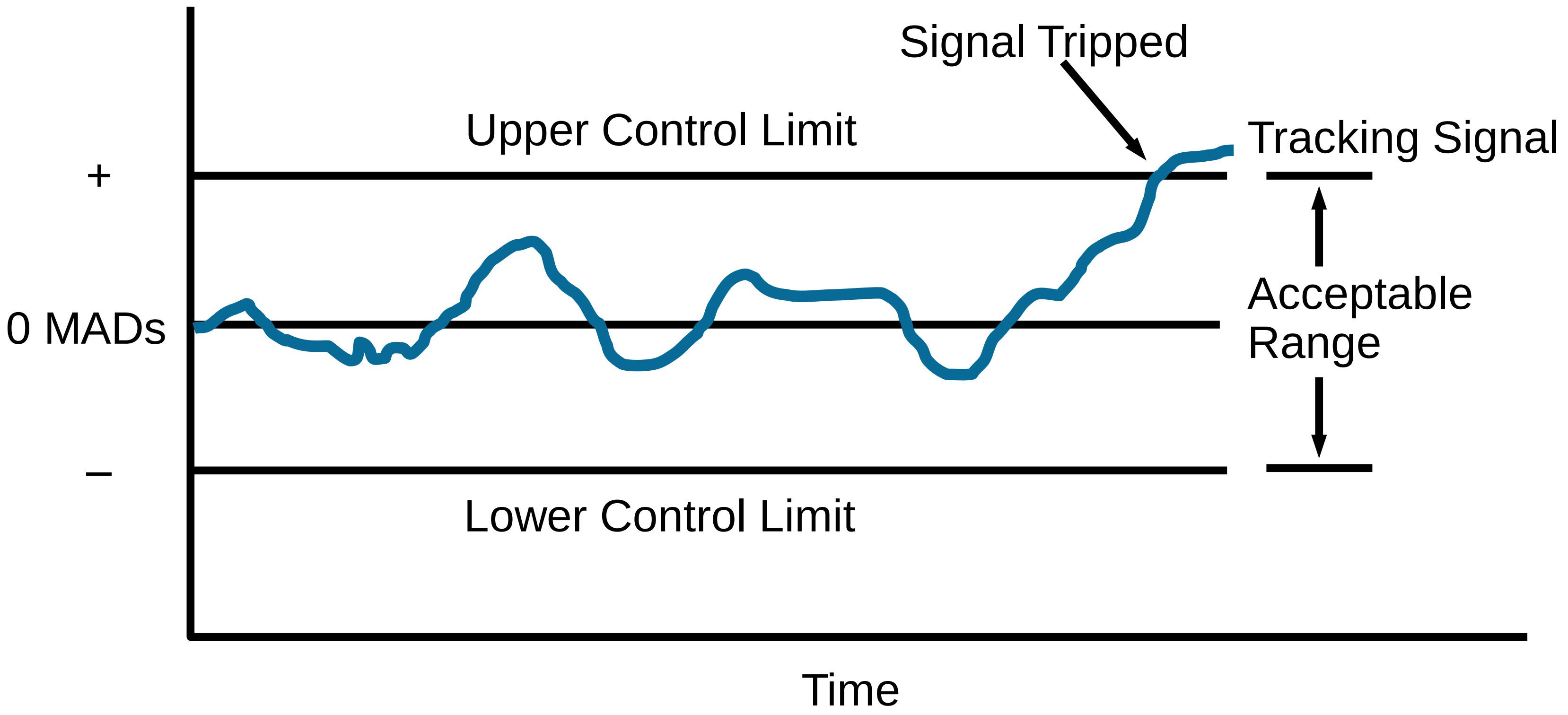
$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n}$$

Monitoring and Controlling Forecasts

- Positive tracking signals indicate demand is greater than forecast
- Negative tracking signals indicate demand is less than forecast
- A good forecast will have about as much positive error as negative error
- Problems are indicated when the signal trips either the upper or lower predetermined limits
- Choose reasonable values for the limits

Monitoring and Controlling Forecasts

FIGURE 5.7 – Plot of Tracking Signals



Kimball’s Bakery Example

- Quarterly sales of croissants (in thousands)

TIME PERIOD	FORECAST DEMAND	ACTUAL DEMAND	ERROR	RSFE	|FORECAST ERROR|	CUMULATIVE ERROR	MAD	TRACKING SIGNAL
1	100	90	–10	–10	10	10	10.0	–1
2	100	95	–5	–15	5	15	7.5	–2
3	100	115	+15	0	15	30	10.0	0
4	110	100	–10	–10	10	40	10.0	–1
5	110	125	+15	+5	15	55	11.0	+0.5
6	110	140	+30	+35	35	85	14.2	+2.5

For Period 6:

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{85}{6} = 14.2$$

$$\text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}} = \frac{35}{14.2} = 2.5 \text{ MADs}$$

Adaptive Smoothing

- Computer monitoring of tracking signals and self-adjustment if a limit is tripped
- In exponential smoothing, the values of α and β are adjusted when the computer detects an excessive amount of variation

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