



CHAPTER

5

Forecasting

To accompany
Quantitative Analysis for Management, Twelfth Edition,
by Render, Stair, Hanna and Hale
Power Point slides created by Jeff Heyl

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LEARNING OBJECTIVES

After completing this chapter, students will be able to:

1. Understand and know when to use various families of forecasting models.
2. Compare moving averages, exponential smoothing, and other time-series models.
3. Seasonally adjust data.
4. Understand Delphi and other qualitative decision-making approaches.
5. Compute a variety of error measures.

CHAPTER OUTLINE

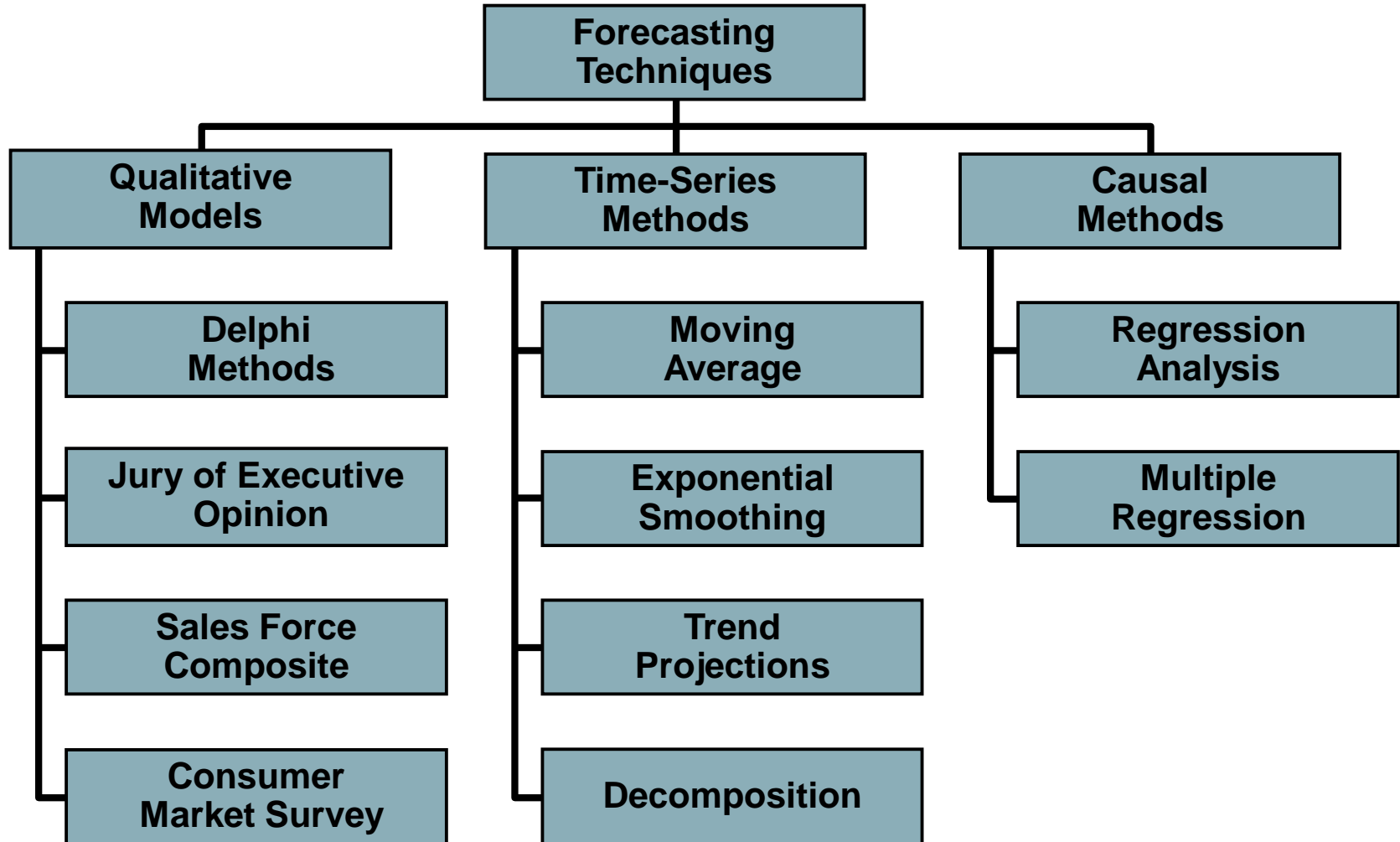
- 5.1 Introduction
- 5.2 Types of Forecasting Models
- 5.3 Components of a Time Series
- 5.4 Measures of Forecast Accuracy
- 5.5 Forecasting Models – Random Variations Only
- 5.6 Forecasting Models – Trend and Random Variations
- 5.7 Adjusting for Seasonal Variations
- 5.8 Forecasting Models – Trend, Seasonal, and Random Variations
- 5.9 Monitoring and Controlling Forecasts

Introduction

- Main purpose of forecasting
 - Reduce uncertainty and make better estimates of what will happen in the future
- Subjective methods
 - Seat-of-the pants methods, intuition, experience
- More formal quantitative and qualitative techniques

Forecasting Models

FIGURE 5.1



Qualitative Models

- Incorporate judgmental or subjective factors
 - Useful when subjective factors are important or accurate quantitative data is difficult to obtain
- Common qualitative techniques
 1. *Delphi method*
 2. *Jury of executive opinion*
 3. *Sales force composite*
 4. *Consumer market surveys*

Qualitative Models

- *Delphi Method*
 - Iterative group process
 - Respondents provide input to decision makers
 - Repeated until consensus is reached
- *Jury of Executive Opinion*
 - Collects opinions of a small group of high-level managers
 - May use statistical models for analysis

Qualitative Models

- *Sales Force Composite*
 - Allows individual salespersons estimates
 - Reviewed for reasonableness
 - Data is compiled at a district or national level
- *Consumer Market Survey*
 - Information on purchasing plans solicited from customers or potential customers
 - Used in forecasting, product design, new product planning

Time-Series Models

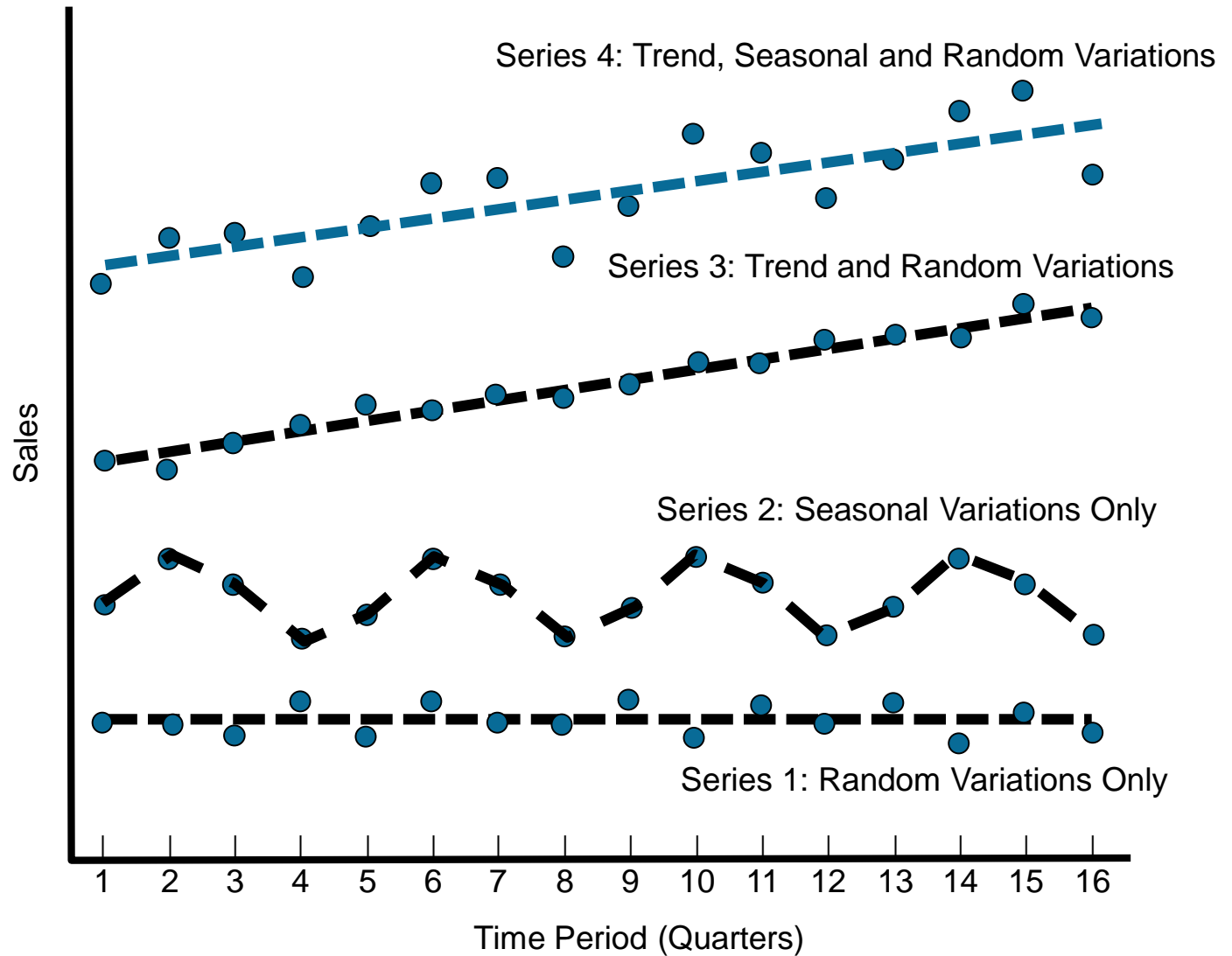
- Predict the future based on the past
- Uses only historical data on one variable
- Extrapolations of past values of a series
- Ignores factors such as
 - Economy
 - Competition
 - Selling price

Components of a Time Series

- Sequence of values recorded at successive intervals of time
- Four possible components
 - Trend (T)
 - Seasonal (S)
 - Cyclical (C)
 - Random (R)

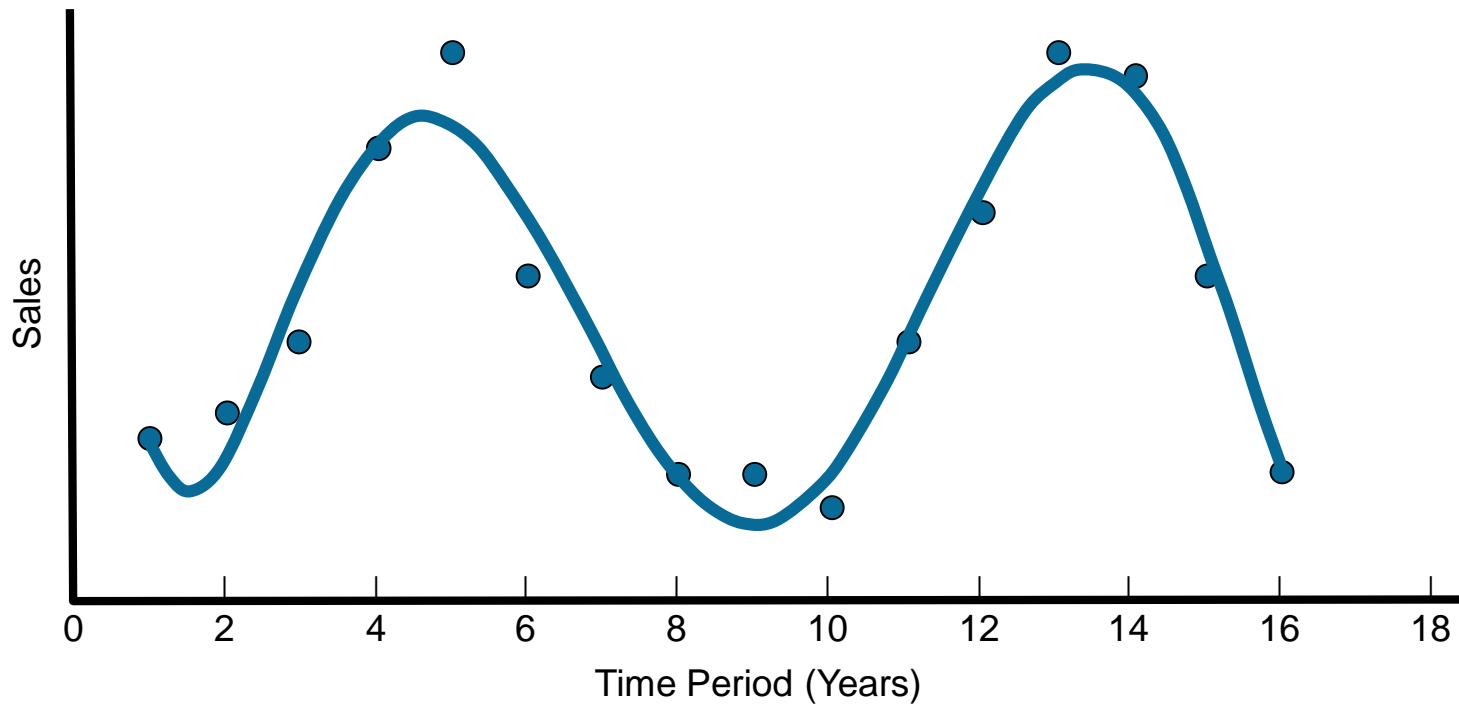
Components of a Time Series

FIGURE 5.2 –
Scatter Diagram for
Four Time Series
of Quarterly Data



Components of a Time Series

FIGURE 5.3 – Scatter Diagram of Times Series with Cyclical and Random Components



Time-Series Models

- Two basic forms
 - Multiplicative

$$\text{Demand} = T \times S \times C \times R$$

- Additive

$$\text{Demand} = T + S + C + R$$

- Combinations are possible

Measures of Forecast Accuracy

- Compare forecasted values with actual values
 - See how well one model works
 - To compare models

Forecast error = Actual value – Forecast value

- Measure of accuracy
 - **Mean absolute deviation (MAD):**

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n}$$

Measures of Forecast Accuracy

TABLE 5.1 – Computing the Mean Absolute Deviation (MAD)

YEAR	ACTUAL SALES OF WIRELESS SPEAKERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST)
1	110	—	
2	100	110	
3	120	100	
4	140	120	
5	170	140	
6	150	170	
7	160	150	
8	190	160	
9	200	190	
10	190	200	
11	—	190	

Measures of Forecast Accuracy

TABLE 5.1 – Computing the Mean Absolute Deviation (MAD)

YEAR	ACTUAL SALES OF WIRELESS SPEAKERS	FORECAST SALES
1	110	—
2	100	110
3	120	100
4	140	120
5	170	140
6	150	170
7	160	150
8	190	160
9	200	190
10	190	200
11	—	190

- Forecast based on naïve model
- No attempt to adjust for time series components

Accuracy

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{160}{9} = 17.8$$

YEAR	ACTUAL SALES OF WIRELESS SPEAKERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL - FORECAST)
1	110	—	—
2	100	110	$ 100 - 110 = 10$
3	120	100	$ 120 - 110 = 20$
4	140	120	$ 140 - 120 = 20$
5	170	140	$ 170 - 140 = 30$
6	150	170	$ 150 - 170 = 20$
7	160	150	$ 160 - 150 = 10$
8	190	160	$ 190 - 160 = 30$
9	200	190	$ 200 - 190 = 10$
10	190	200	$ 190 - 200 = 10$
11	—	190	—
			Sum of errors = 160
			MAD = $160/9 = 17.8$

Measures of Forecast Accuracy

- Other common measures
 - **Mean squared error (MSE)**

$$\text{MSE} = \frac{\sum (\text{error})^2}{n}$$

- **Mean absolute percent error (MAPE)**

$$\text{MAPE} = \frac{\sum \left| \frac{\text{error}}{\text{actual}} \right|}{n} 100\%$$

- **Bias** is the average error

Forecasting Random Variations

- No other components are present
- Averaging techniques smooth out forecasts
 - Moving averages
 - Weighted moving averages
 - Exponential smoothing

Moving Averages

- Used when demand is relatively steady over time
 - The next forecast is the average of the most recent n data values from the time series
 - Smooths out short-term irregularities in the data series

$$\text{Moving average forecast} = \frac{\text{Sum of demands in previous } n \text{ periods}}{n}$$

Moving Averages

- Mathematically

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \dots + Y_{t-n+1}}{n}$$

where

F_{t+1} = forecast for time period $t + 1$

Y_t = actual value in time period t

n = number of periods to average

Wallace Garden Supply

- Wallace Garden Supply wants to forecast demand for its Storage Shed
 - Collected data for the past year
 - Use a three-month moving average ($n = 3$)

Wallace Garden Supply

TABLE 5.2

MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11.67$
May	19	$(12 + 13 + 16)/3 = 13.67$
June	23	$(13 + 16 + 19)/3 = 16.00$
July	26	$(16 + 19 + 23)/3 = 19.33$
August	30	$(19 + 23 + 26)/3 = 22.67$
September	28	$(23 + 26 + 30)/3 = 26.33$
October	18	$(26 + 30 + 28)/3 = 28.00$
November	16	$(30 + 28 + 18)/3 = 25.33$
December	14	$(28 + 18 + 16)/3 = 20.67$
January	—	$(18 + 16 + 14)/3 = 16.00$

Weighted Moving Averages

- *Weighted moving averages* use weights to put more emphasis on previous periods
 - Often used when a trend or other pattern is emerging

$$F_{t+1} = \frac{\hat{a}(\text{Weight in period } i)(\text{Actual value in period})}{\hat{a}(\text{Weights})}$$

- Mathematically

$$F_{t+1} = \frac{w_1 Y_t + w_2 Y_{t-1} + \dots + w_n Y_{t-n+1}}{w_1 + w_2 + \dots + w_n}$$

where

w_i = weight for the i^{th} observation

Wallace Garden Supply

- Use a 3-month weighted moving average model to forecast demand
 - Weighting scheme

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
$\textcircled{3} \times \text{Sales last month} + \textcircled{2} \times \text{Sales two months ago} + \textcircled{1} \times \text{Sales three months ago}$	
<hr/> $\textcircled{6}$	
<hr/> Sum of the weights	

Wallace Garden Supply

TABLE 5.3

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12.17$
May	19	$[(3 \times 16) + (2 \times 13) + (12)]/6 = 14.33$
June	23	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17.00$
July	26	$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20.50$
August	30	$[(3 \times 26) + (2 \times 23) + (19)]/6 = 23.83$
September	28	$[(3 \times 30) + (2 \times 26) + (23)]/6 = 27.50$
October	18	$[(3 \times 28) + (2 \times 30) + (26)]/6 = 28.33$
November	16	$[(3 \times 18) + (2 \times 28) + (30)]/6 = 23.33$
December	14	$[(3 \times 16) + (2 \times 18) + (28)]/6 = 18.67$
January	—	$[(3 \times 14) + (2 \times 16) + (18)]/6 = 15.33$

Exponential Smoothing

- **Exponential smoothing**
 - A type of moving average
 - Easy to use
 - Requires little record keeping of data

New forecast = Last period's forecast
+ α (Last period's actual demand
– Last period's forecast)

α is a weight (or **smoothing constant**) with a value $0 \leq \alpha \leq 1$

Exponential Smoothing

- Mathematically

$$F_{t+1} = F_t + \alpha(Y_t - F_t)$$

where

F_{t+1} = new forecast (for time period $t + 1$)

F_t = previous forecast (for time period t)

α = smoothing constant ($0 \leq \alpha \leq 1$)

Y_t = previous period's actual demand

The idea is simple – the new estimate is the old estimate plus some fraction of the error in the last period

Exponential Smoothing Example

- In January, February's demand for a certain car model was predicted to be 142
- Actual February demand was 153 autos
- Using a smoothing constant of $\alpha = 0.20$, what is the forecast for March?

$$\begin{aligned}\text{New forecast (for March demand)} &= 142 + 0.2(153 - 142) \\ &= 144.2 \text{ or } 144 \text{ autos}\end{aligned}$$

- If actual March demand = 136

$$\begin{aligned}\text{New forecast (for April demand)} &= 144.2 + 0.2(136 - 144.2) \\ &= 142.6 \text{ or } 143 \text{ autos}\end{aligned}$$

Selecting the Smoothing Constant

- Selecting the appropriate value for α is key to obtaining a good forecast
- The objective is always to generate an accurate forecast
- The general approach is to develop trial forecasts with different values of α and select the α that results in the lowest MAD

Port of Baltimore Example

TABLE 5.4 – Exponential Smoothing Forecast for $\alpha = 0.1$ and $\alpha = 0.5$

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST USING $\alpha = 0.10$	FORECAST USING $\alpha = 0.50$
1	180	175	175
2	168	$175.5 = 175.00 + 0.10(180 - 175)$	177.5
3	159	$174.75 = 175.50 + 0.10(168 - 175.50)$	172.75
4	175	$173.18 = 174.75 + 0.10(159 - 174.75)$	165.88
5	190	$173.36 = 173.18 + 0.10(175 - 173.18)$	170.44
6	205	$175.02 = 173.36 + 0.10(190 - 173.36)$	180.22
7	180	$178.02 = 175.02 + 0.10(205 - 175.02)$	192.61
8	182	$178.22 = 178.02 + 0.10(180 - 178.02)$	186.30
9	?	$178.60 = 178.22 + 0.10(182 - 178.22)$	184.15

Port of Baltimore Example

TABLE 5.5 – Absolute Deviations and MADs

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = 0.10$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.10$	FORECAST WITH $\alpha = 0.50$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.50$
1	180	175	5	175	5
2	168	175.5	7.5	177.5	9.5
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.3
Sum of absolute deviations			82.45		98.63
MAD = $\frac{\Sigma \text{deviations} }{n}$ =			10.31		MAD = 12.33

Best choice

Using Software

PROGRAM 5.1A – Selecting the Forecasting Model

The screenshot shows the Excel QM software interface. The 'Excel QM' tab is selected in the top ribbon. The 'Forecasting' menu is open, showing a list of models. A callout bubble points to the 'Forecasting' menu item with the text: 'Click Alphabetical and scroll down to Forecasting.' Another callout bubble points to the 'Excel QM' tab with the text: 'Click the Excel QM Tab.' A third callout bubble points to the 'Weighted Moving Average' model in the forecasting list with the text: 'With cursor on Forecasting, a menu appears to the right. Select the model you want and an initialization window will open.'

FILE HOME INSERT PAGE LAYOUT FORMULAS DATA REVIEW VIEW Excel QM

0 1 2 Gridlines Headings Instruction

Assignment
Breakeven Analysis
Decision Analysis
Forecasting
Inventory
Linear, Integer & Mixed Integer Programming
Markov Chains
Material Requirements Planning
Network Analysis
Network Analysis as LP
Project Management
Quality Control
Simulation
Statistics (mean, var, sd; Normal Dist)
Transportation
Waiting Lines
Display OM Models Only
Display QM Models Only
Display All Models

By Alpha chapter Menus

Weighted Moving Average
Exponential Smoothing
Trend Adjusted Exponential Smoothing
Regression/Trend Analysis
Multiple Regression
Multiplicative Seasonal Model
Regression Projector
Error Analysis

Click the Excel QM Tab.

Click Alphabetical and scroll down to Forecasting.

With cursor on Forecasting, a menu appears to the right. Select the model you want and an initialization window will open.

Using Software

PROGRAM 5.1B – Initializing Excel QM

The image shows a 'Spreadsheet Initialization' dialog box with the following fields and options:

- Title:** Wallace Garden Supply (Callout: Enter a title.)
- Number of (past) periods of data:** 12 (Callout: Enter the number of past periods of data.)
- Name for period:** Period (Callout: Enter the number of periods to be averaged.)
- Number of periods to average:** 3 (Callout: Enter the number of periods to be averaged.)
- Sheet name:** (empty)
- Options:**
 - Tracking Signal
 - Graph

Buttons at the bottom: Use Default Settings, Help, Cancel, and OK (Callout: Click OK.).

Using Software

PROGRAM 5.1C – Excel QM Output

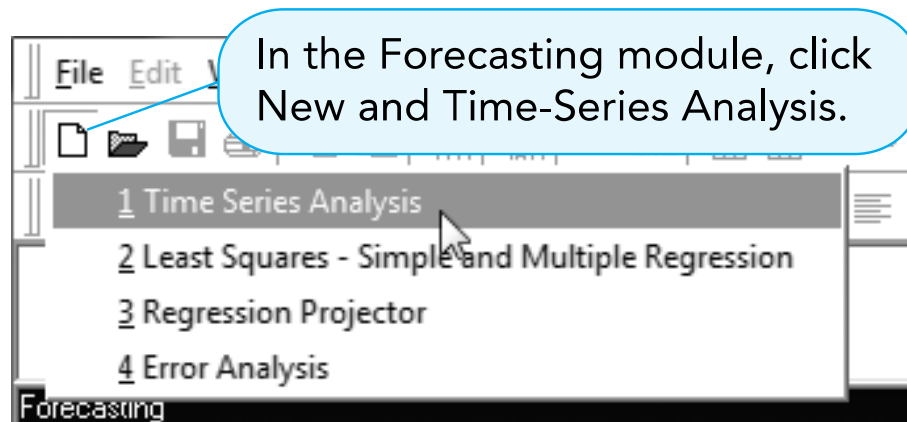
	A	B	C						
1	Wallace Garden Supl			Enter the demand data and the weights. The calculations will automatically be performed.					
2									
3	Forecasting			Weighted moving averages - 3 period moving average					
4									
5	Enter the data in the shaded area. Enter weights in INCREASING order from top to bottom								
6									
7	Data			Forecasts and Error Analysis					
8	Period	Demand	Weights	Forecast	Error	Absolute	Squared	Abs Pct Err	
9	Period 1	10	1						
10	Period 2	12	2						
11	Period 3	13	3						
12	Period 4	16		12.16667	3.833333	3.833333	14.69444	23.96%	
13	Period 5	19		14.33333	4.666667	4.666667	21.77778	24.56%	
14	Period 6	23		17	6	6	36	26.09%	
15	Period 7	26		20.5	5.5	5.5	30.25	21.15%	
16	Period 8	30		23.83333	6.166667	6.166667	38.02778	20.56%	
17	Period 9	28							
18	Period 10	18							
19	Period 11	16		23.33333	-7.33333	7.333333	53.77778	45.83%	
20	Period 12	14		18.66667	-4.66667	4.666667	21.77778	33.33%	
21				4.333333		49	323.3333	254.68%	
22				0.481481	5.444444	35.92593	28.30%		
23				Bias	MAD	MSE	MAPE		
24					SE	6.796358			
25	Next period	15.333333							

The measures of accuracy are shown here.

The forecast for the next period is here.

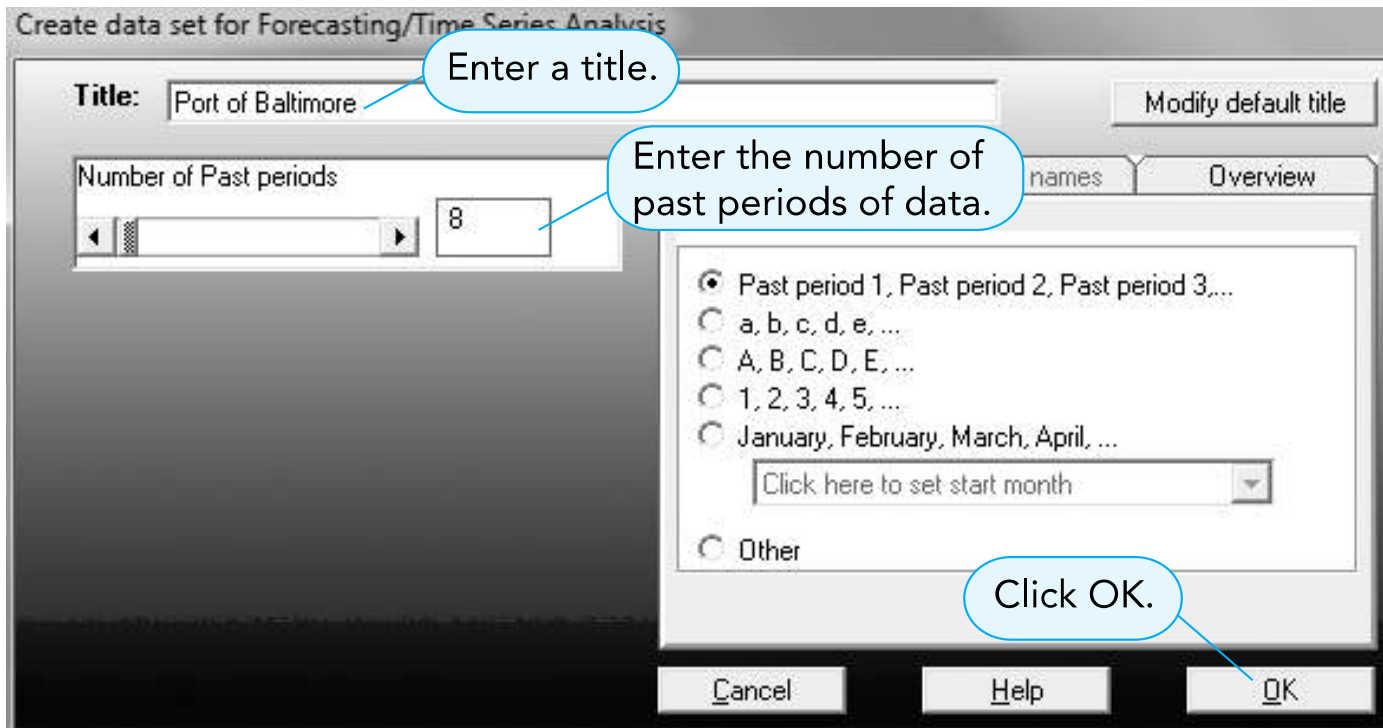
Using Software

PROGRAM 5.2A – Selecting Time-Series Analysis in QM for Windows



Using Software

PROGRAM 5.2B – Entering Data



Using Software

PROGRAM 5.2C – Selecting the Model and Entering Data

Method

Alpha for smoothing

Exponential Smoothing

Weighted Moving Averages

Exponential Smoothing

Exponential Smoothing with trend

Enter the value for the smoothing constant.

Click here to see the models. Other input areas appear based on the model. Select Exponential Smoothing and a window opens for you to enter the smoothing constant.

Past period			Forecast
3	159	0	0
4	175	0	0
5	190	0	0
6	205	0	0
7	220	0	0
8		0	0

Enter the data. Then click Solve at the top of the page.

Using Software

PROGRAM 5.2D – Output for Port of Baltimore Example

The screenshot shows a software interface with a menu bar (File, Edit, View, Module, Format, Tools, Window, Help) and a toolbar. The 'Window' menu is open, showing options: Cascade, Tile, Edit Data, 1 Forecasting Results, 2 Details and Error Analysis (highlighted), 3 Errors as a function of alpha, 4 Control (Tracking Signal), and 5 Graph. A callout bubble points to the 'Window' menu with the text: 'Additional output is available under Window.'

The main window displays 'Exponential Smoothing' as the method. Below it, a table shows error measures and a forecast for the next period. A callout bubble points to the error measures table with the text: 'The measures of accuracy are shown here.'

Another callout bubble points to the 'next period' forecast value with the text: 'The forecast for next period is here.'

Measure	
Error Measures	
Bias (Mean Error)	1.069
MAD (Mean Absolute Deviation)	10.868
MSE (Mean Squared Error)	207.24
Standard Error (denom= $n-2=5$)	17.033
MAPE (Mean Absolute Percent Error)	5.999%
Forecast	
next period	180.748

Port of Baltimore Summary

Forecasting – Trend and Random

- Exponential smoothing does not respond to trends
- A more complex model can be used
- The basic approach
 - Develop an exponential smoothing forecast
 - Adjust it for the trend

$$\text{Forecast including trend } (FIT_{t+1}) = \text{Smoothed forecast } (F_{t+1}) + \text{Smoothed Trend } (T_{t+1})$$

Exponential Smoothing with Trend

- The equation for the trend correction uses a new smoothing constant β
- F_t and T_t must be given or estimated
- Three steps in developing FIT_t

Step 1: Compute smoothed forecast F_{t+1}

Smoothed forecast = Previous forecast including trend + α (Last error)

$$F_{t+1} = FIT_t + a(Y_t - FIT_t)$$

Exponential Smoothing with Trend

Step 2: Update the trend (T_{t+1}) using

Smoothed forecast = Previous forecast including trend + β (Error or excess in trend)

$$T_{t+1} = T_t + b(F_{t+1} - FIT_t)$$

Step 3: Calculate the trend-adjusted exponential smoothing forecast (FIT_{t+1}) using

Forecast including trend (FIT_{t+1}) = Smoothed forecast (F_{t+1}) + Smoothed trend (T_{t+1})

$$FIT_{t+1} = F_{t+1} + T_{t+1}$$

Selecting a Smoothing Constant

- A high value of β makes the forecast more responsive to changes in trend
- A low value of β gives less weight to the recent trend and tends to smooth out the trend
- Values are often selected using a trial-and-error approach based on the value of the MAD for different values of β

Midwestern Manufacturing

- Demand for electrical generators from 2007 – 2013
 - Midwest assumes F_1 is perfect, $T_1 = 0$, $\alpha = 0.3$, $\beta = 0.4$

$$FIT_1 = F_1 + T_1 = 74 + 0 = 74$$

TABLE 5.6 –
Demand

YEAR	ELECTRICAL GENERATORS SOLD
2007	74
2008	79
2009	80
2010	90
2011	105
2012	142
2013	122

Midwestern Manufacturing

For 2008 (time period 2)

Step 1: Compute F_{t+1}

$$\begin{aligned} F_2 &= FIT_1 + \alpha(Y_1 - FIT_1) \\ &= 74 + 0.3(74 - 74) = 74 \end{aligned}$$

Step 2: Update the trend

$$\begin{aligned} T_2 &= T_1 + \beta(F_2 - FIT_1) \\ &= 0 + .4(74 - 74) = 0 \end{aligned}$$

Midwestern Manufacturing

Step 3: Calculate the trend-adjusted exponential smoothing forecast (F_{t+1}) using

$$\begin{aligned}FIT_2 &= F_2 + T_2 \\ &= 74 + 0 = 74\end{aligned}$$

Midwestern Manufacturing

For 2009 (time period 3)

Step 1:
$$F_3 = FIT_2 + \alpha(Y_2 - FIT_2)$$
$$= 74 + 0.3(79 - 74) = 75.5$$

Step 2:
$$T_3 = T_2 + .4(F_3 - FIT_2)$$
$$= 0 + .4(75.5 - 74) = 0.6$$

Step 3:
$$FIT_3 = F_3 + T_3$$
$$= 75.5 + 0.6 = 76.1$$

Midwestern Manufacturing

TABLE 5.7 – Exponential Smoothing with Trend Forecasts

TIME (<i>t</i>)	DEMAND (<i>Y_t</i>)	$F_{t+1} = FIT_t + 0.3(Y_t - FIT_t)$	$T_{t+1} = T_t + 0.4(F_{t+1} - FIT_t)$	$FIT_{t+1} = F_{t+1} + T_{t+1}$
1	74	74	0	74
2	79	74 $= 74 + 0.3(74 - 74)$	0 $= 0 + 0.4(74 - 74)$	74 $= 74 + 0$
3	80	75.5 $= 74 + 0.3(79 - 74)$	0.6 $= 0 + 0.4(75.5 - 74)$	76.1 $= 75.5 + 0.6$
4	90	77.270 $= 76.1 + 0.3(80 - 76.1)$	1.068 $= 0.6 + 0.4(77.27 - 76.1)$	78.338 $= 77.270 + 1.068$
5	105	81.837 $= 78.338 + 0.3(90 - 78.338)$	2.468 $= 1.068 + 0.4(81.837 - 78.338)$	84.305 $= 81.837 + 2.468$
6	142	90.514 $= 84.305 + 0.3(105 - 84.305)$	4.952 $= 2.468 + 0.4(90.514 - 84.305)$	95.466 $= 90.514 + 4.952$
7	122	109.426 $= 95.446 + 0.3(142 - 95.466)$	10.536 $= 4.952 + 0.4(109.426 - 95.466)$	119.962 $= 109.426 + 10.536$
8		120.573 $= 119.962 + 0.3(122 - 119.962)$	10.780 $= 10.536 + 0.4(120.573 - 119.962)$	131.353 $= 120.573 + 10.780$

Midwestern Manufacturing

PROGRAM 5.3 – Output from Excel QM Trend-Adjusted Exponential Smoothing

	A	B	C	D	E	F	G	H	I	J
1	Midwestern Manufacturing Company Example									
2										
3	Forecasting	Trend adjusted exponential smoothing								
4	Enter alpha and beta (between 0 and 1), enter the past demands in the shaded column then enter a starting forecast. If the starting forecast is not in the first period then delete the error analysis for									
5										
6										
7	Alpha	0.3								
8	Beta	0.4								
9	Data		Forecasts and Error Analysis							
10	Period	Demand	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t	Error	Absolute	Squared	Abs Pct Err	
11	Period 1	74	74		74	0	0	0	00.00%	
12	Period 2	79	74	0	74	5	5	25	06.33%	
13	Period 3	80	75.5	0.6	76.1	3.9	3.9	15.21	04.88%	
14	Period 4	90	77.27	1.068	78.338	11.662	11.662	136.0022	12.96%	
15	Period 5	105	81.8366	2.46744	84.30404	20.69596	20.696	428.3228	19.71%	
16	Period 6	142	90.512828	4.950955	95.4637832	46.53622	46.5362	2165.619	32.77%	
17	Period 7	122	109.4246482	10.5353	119.959949	2.040051	2.04005	4.161806	0.016722	
18		Next period	120.5719646	10.78011	131.352072					
19			Total			89.83423	89.8342	2774.316	78.32%	
20			Average			12.83346	12.8335	396.3309	11.19%	
21						Bias	MAD	MSE	MAPE	
22							SE	23.55554		

The forecast for next period is here.

Trend Projections

- Fits a trend line to a series of historical data points
- Projected into the future for medium- to long-range forecasts
- Trend equations can be developed based on exponential or quadratic models
- Linear model developed using regression analysis is simplest

Trend Projections

- Mathematical formula

$$\hat{Y} = b_0 + b_1X$$

where

\hat{Y} = predicted value

b_0 = intercept

b_1 = slope of the line

X = time period (i.e., $X = 1, 2, 3, \dots, n$)

Midwestern Manufacturing

- Based on least squares regression, the forecast equation is

$$\hat{Y} = 56.71 + 10.54X$$

- Year 2014 is coded as $X = 8$

$$\begin{aligned}(\text{sales in 2014}) &= 56.71 + 10.54(8) \\ &= 141.03, \text{ or } 141 \text{ generators}\end{aligned}$$

- For $X = 9$

$$\begin{aligned}(\text{sales in 2015}) &= 56.71 + 10.54(9) \\ &= 151.57, \text{ or } 152 \text{ generators}\end{aligned}$$

Midwestern Manufacturing

PROGRAM 5.4 – Output from Excel QM for Trend Line

	A	B	C	D	E	F	G	H	I
1	Midwestern Manufacturing Company Example								
2									
3	Forecasting		Simple linear regression						
4	<div style="border: 1px solid black; padding: 5px;"> If this is trend analysis then simply enter the past demands in the demand column. If this is causal regression then enter the y,x pairs with y first and enter a new value of x at the bottom in order to </div>								
7									
8	Data			Forecasts and Error Analysis					
9	Period	Demand (y)	Period(x)	Forecast	Error	Absolute	Squared	Abs Pct Err	
10	Period 1	74	1	67.25	6.75	6.75	45.5625	09.12%	
11	Period 2	79	2	77.7857	1.2143	1.2143	1.4745	01.54%	
12	Period 3	80	3					10.40%	
13	Period 4	90	4					09.84%	
14	Period 5	105	5					04.18%	
15	Period 6	142	6					15.54%	
16	Period 7	122	7	130.4643	-8.4643	8.4643	71.6441	06.94%	
17				Total	-4.3E-14	60.0714	772.8214	57.57%	
18	Intercept	56.7143		Average	-6.1E-15	8.5816	110.4031	08.22%	
19	Slope	10.5357			Bias	MAD	MSE	MAPE	
20						SE	12.4324		
21	Forecast	141	8						
22						Correlati	0.89491		
23						cient of determination	0.800863		

To forecast other time periods, enter the time period here.

The forecast for next period is here.

Midwestern Manufacturing

PROGRAM 5.5 – Output from QM for Trend Line

Forecasting Results

Midwestern Manufacturing Company Summary

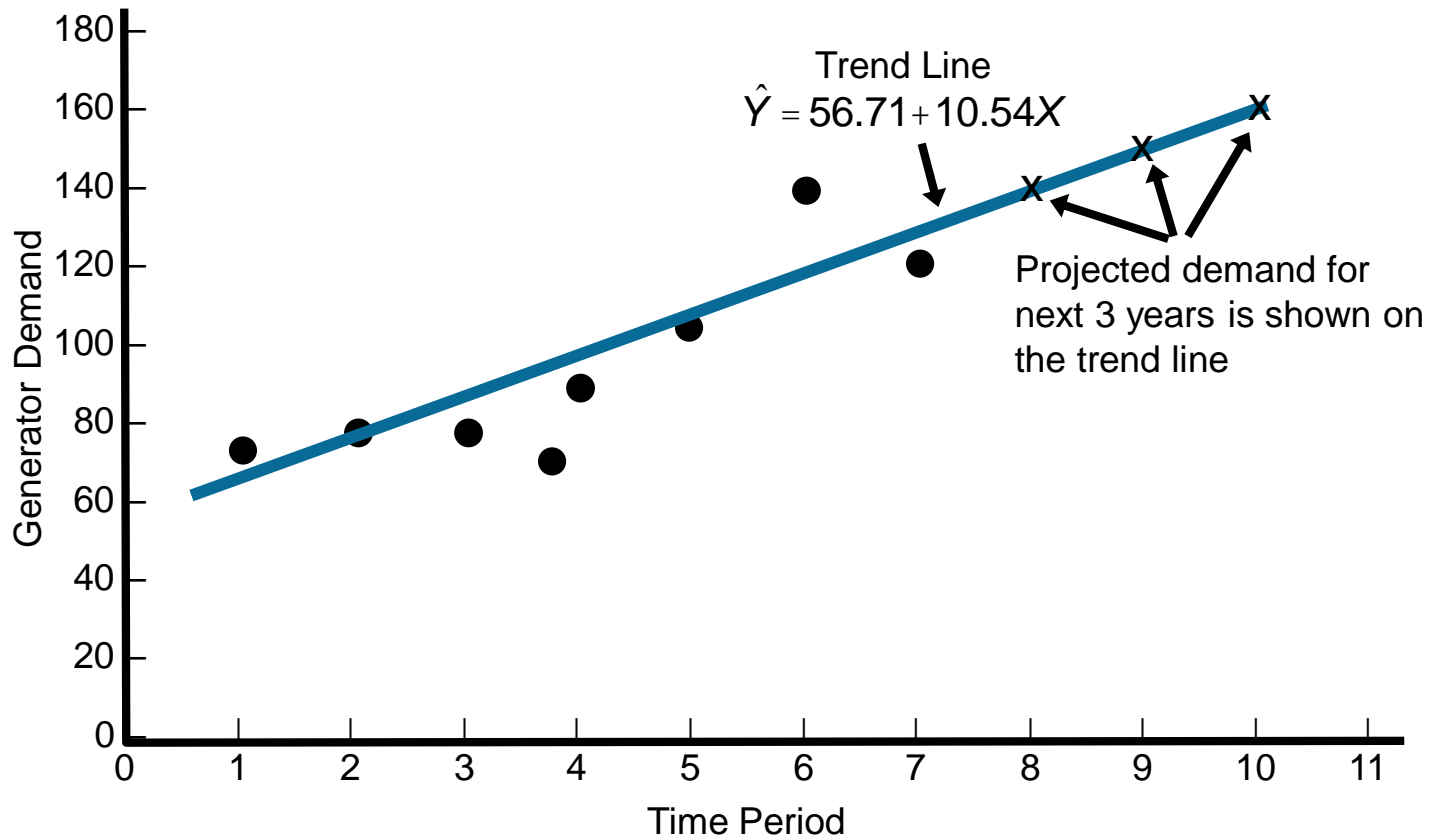
Measure		Future Period	Forecast
Error Measures		8	141
Bias (Mean Error)	0	9	151.536
MAD (Mean Absolute Deviation)	8.582	10	162.071
MSE (Mean Squared Error)	110.403	11	172.607
Standard Error (denom=n-2=5)	12.432	12	183.143
MAPE (Mean Absolute Percent	8.224%	13	193.679
Regression line		14	204.214
Demand(y) = 56.714		15	214.75
+ 10.536 * Time(x)		16	225.286
Statistics			235.822
Correlation coefficient			246.357
Coefficient of determination (r ²)			256.893
		20	267.429
		21	277.964

Forecasts for future time periods are shown here.

The trend line is shown over two lines.

Midwestern Manufacturing

FIGURE 5.4 – Generator Demand Based on Trend Line



Seasonal Variations

- Recurring variations over time may indicate the need for *seasonal* adjustments in the trend line
- A **seasonal index** indicates how a particular season compares with an average season
 - An index of 1 indicates an average season
 - An index > 1 indicates the season is higher than average
 - An index < 1 indicates a season lower than average

Seasonal Indices

- **Deseasonalized data** is created by dividing each observation by the appropriate seasonal index
- Once deseasonalized forecasts have been developed, values are multiplied by the seasonal indices
- Computed in two ways
 - Overall average
 - Centered-moving-average approach

Seasonal Indices with No Trend

- Divide average value for each season by the average of all data
 - Telephone answering machines at Eichler Supplies
 - Sales data for the past two years for one model
 - Create a forecast that includes seasonality

Seasonal Indices with No Trend

TABLE 5.8 – Answering Machine Sales and Seasonal Indices

MONTH	SALES DEMAND		AVERAGE 2- YEAR DEMAND	MONTHLY DEMAND	AVERAGE SEASONAL INDEX
	YEAR 1	YEAR 2			
January	80	100	90	94	0.957
February	85	75	80	94	0.851
March	80	90	85	94	0.904
April	110	90	100	94	1.064
May	115	131	123	94	1.309
June	120	110	115	94	1.223
July	100	110	105	94	1.117
August	110	90	100	94	1.064
September	85	95	90	94	0.957
October	75	85	80	94	0.851
November	85	75	80	94	0.851
December	80	80	80	94	0.851

Total average demand = $\frac{1,128}{12} = 94$

Average monthly demand = $\frac{1,128}{12 \text{ months}} = 94$

Seasonal index = $\frac{\text{Average 2-year demand}}{\text{Average monthly demand}}$

Seasonal Indices with No Trend

- Calculations for the seasonal indices

$$\text{Jan.} \quad \frac{1,200}{12} \cdot 0.957 = 96$$

$$\text{Feb.} \quad \frac{1,200}{12} \cdot 0.851 = 85$$

$$\text{Mar.} \quad \frac{1,200}{12} \cdot 0.904 = 90$$

$$\text{Apr.} \quad \frac{1,200}{12} \cdot 1.064 = 106$$

$$\text{May} \quad \frac{1,200}{12} \cdot 1.309 = 131$$

$$\text{June} \quad \frac{1,200}{12} \cdot 1.223 = 122$$

$$\text{July} \quad \frac{1,200}{12} \cdot 1.117 = 112$$

$$\text{Aug.} \quad \frac{1,200}{12} \cdot 1.064 = 106$$

$$\text{Sept.} \quad \frac{1,200}{12} \cdot 0.957 = 96$$

$$\text{Oct.} \quad \frac{1,200}{12} \cdot 0.851 = 85$$

$$\text{Nov.} \quad \frac{1,200}{12} \cdot 0.851 = 85$$

$$\text{Dec.} \quad \frac{1,200}{12} \cdot 0.851 = 85$$

Seasonal Indices with Trend

- Changes could be due to trend, seasonal, or random
- **Centered moving average (CMA)** approach prevents trend being interpreted as seasonal
- Turner Industries sales contain both trend and seasonal components

Seasonal Indices with Trend

- Steps in CMA
 1. Compute the CMA for each observation (where possible)
 2. Compute the seasonal ratio = $\text{Observation}/\text{CMA}$ for that observation
 3. Average seasonal ratios to get seasonal indices
 4. If seasonal indices do not add to the number of seasons, multiply each index by $(\text{Number of seasons})/(\text{Sum of indices})$

Turner Industries

TABLE 5.9 – Quarterly Sales Data

QUARTER	YEAR 1	YEAR 2	YEAR 3	AVERAGE
1	108	116	123	115.67
2	125	134	142	133.67
3	150	159	168	159.00
4	141	152	165	152.67
Average	131.00	140.25	149.50	140.25

Definite trend

Seasonal pattern

Turner Industries

- To calculate the CMA for quarter 3 of year 1, compare the actual sales with an average quarter centered on that time period
- Use 1.5 quarters before quarter 3 and 1.5 quarters after quarter 3
 - Take quarters 2, 3, and 4 and one half of quarters 1, year 1 and quarter 1, year 2

$$\text{CMA}(q3, y1) = \frac{0.5(108) + 125 + 150 + 141 + 0.5(116)}{4} = 132.0$$

Turner Industries

- Compare the actual sales in quarter 3 to the CMA to find the seasonal ratio

$$\text{Seasonal ratio} = \frac{\text{Sales in quarter 3}}{\text{CMA}} = \frac{150}{132.0} = 1.136$$

Turner Industries

TABLE 5.10 – Centered Moving Averages and Seasonal Ratios

YEAR	QUARTER	SALES	CMA	SEASONAL RATIO
1	1	108		
	2	125		
	3	150	132.000	1.136
	4	141	134.125	1.051
2	1	116	136.375	0.851
	2	134	138.875	0.965
	3	159	141.125	1.127
	4	152	143.000	1.063
3	1	123	145.125	0.848
	2	142	147.875	0.960
	3	168		
	4	165		

Turner Industries

- The two seasonal ratios for each quarter are averaged to get the seasonal index

$$\text{Index for quarter 1} = I_1 = (0.851 + 0.848)/2 = 0.85$$

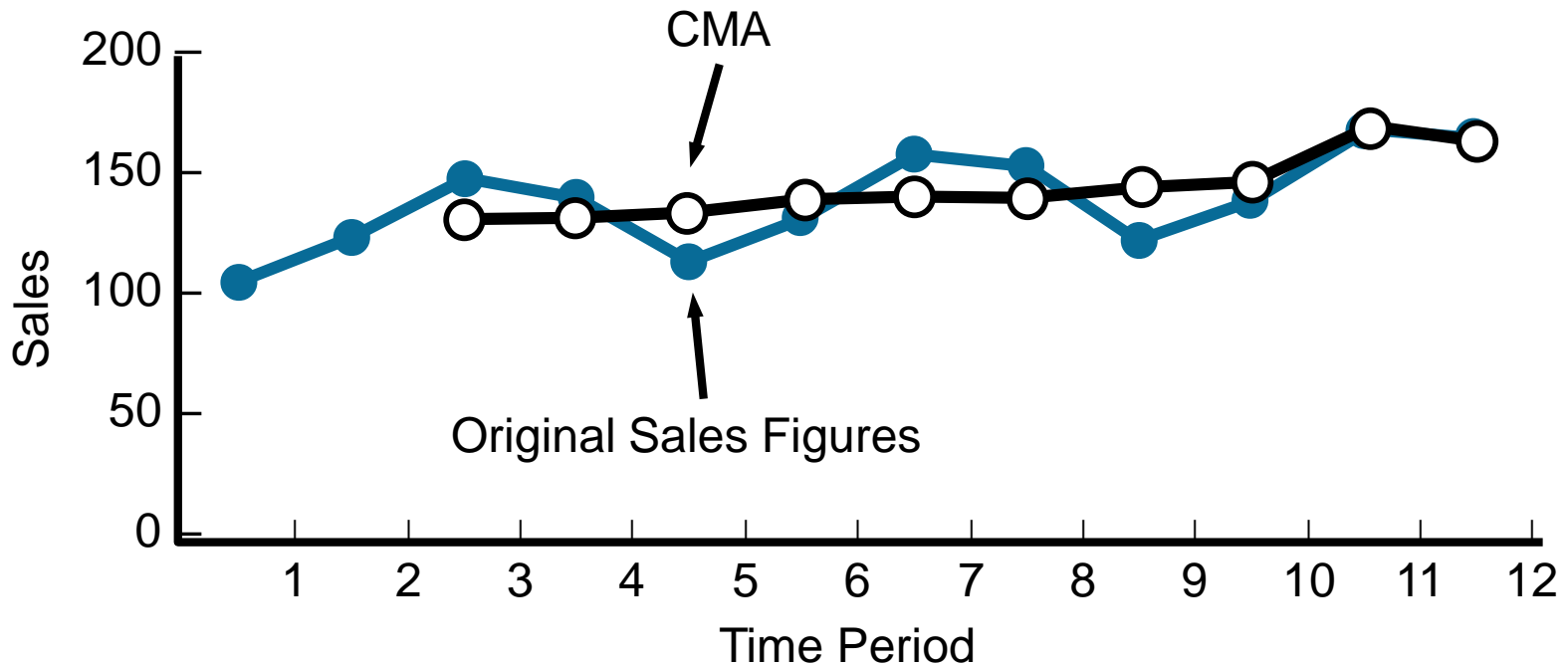
$$\text{Index for quarter 2} = I_2 = (0.965 + 0.960)/2 = 0.96$$

$$\text{Index for quarter 3} = I_3 = (1.136 + 1.127)/2 = 1.13$$

$$\text{Index for quarter 4} = I_4 = (1.051 + 1.063)/2 = 1.06$$

Turner Industries

- Scatterplot of Turner Industries Sales Data and Centered Moving Average



Trend, Seasonal, and Random Variations

- **Decomposition** – isolating linear trend and seasonal factors to develop more accurate forecasts
- Five steps to decomposition
 - Compute seasonal indices using CMAs.
 - Deseasonalize the data by dividing each number by its seasonal index
 - Find the equation of a trend line using the deseasonalized data
 - Forecast for future periods using the trend line
 - Multiply the trend line forecast by the appropriate seasonal index

Deseasonalized Data

TABLE 5.11

SALES (\$1,000,000s)	SEASONAL INDEX	DESEASONALIZED SALES (\$1,000,000s)
108	0.85	127.059
125	0.96	130.208
150	1.13	132.743
141	1.06	133.019
116	0.85	136.471
134	0.96	139.583
159	1.13	140.708
152	1.06	143.396
123	0.85	144.706
142	0.96	147.917
168	1.13	148.673
165	1.06	155.660

Deseasonalized Data

- Find a trend line using the deseasonalized data where $X = \text{time}$

$$b_1 = 2.34 \quad b_0 = 124.78$$

$$\hat{Y} = 124.78 + 2.34X$$

- Develop a forecast for quarter 1, year 4 ($X = 13$) using this trend and multiply the forecast by the appropriate seasonal index

$$\begin{aligned}\hat{Y} &= 124.78 + 2.34(13) \\ &= 155.2 \text{ (before seasonality adjustment)}\end{aligned}$$

Deseasonalized Data

- Find a trend line using the deseasonalized data where

Including the seasonal index

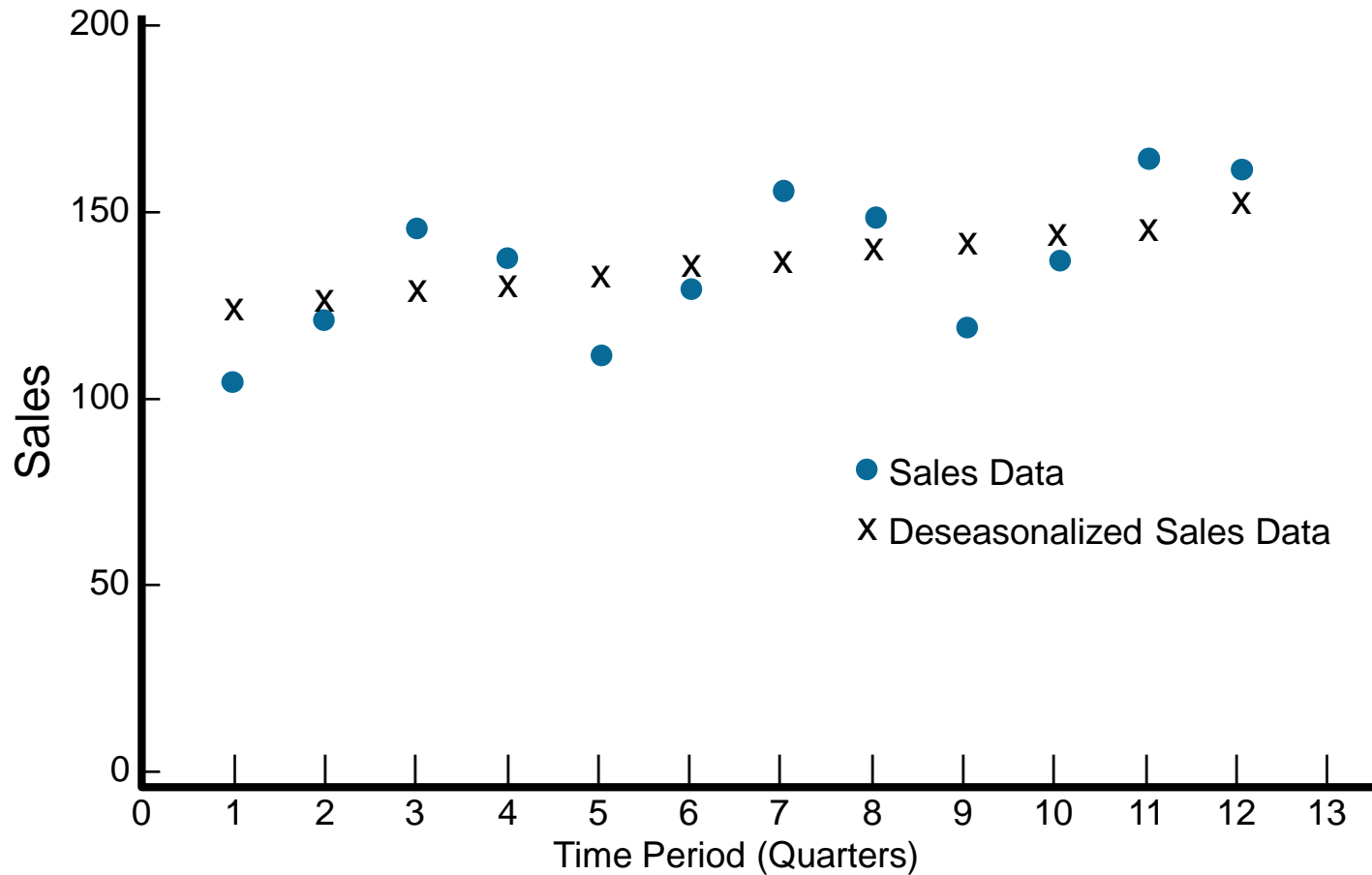
$$\hat{Y} \cdot I_1 = 155.2 \cdot 0.85 = 131.92$$

- Develop a forecast for quarter 1, year 4 ($X = 13$) using this trend and multiply the forecast by the appropriate seasonal index

$$\begin{aligned}\hat{Y} &= 124.78 + 2.34(13) \\ &= 155.2 \text{ (before seasonality)}\end{aligned}$$

Deseasonalized Data

FIGURE 5.5



Using Software

PROGRAM 5.6A – QM for Windows Input

The screenshot shows the 'Turner Industries Sales' input screen in the QM for Windows software. The 'Method' dropdown is set to 'Multiplicative Decomposition (seasonal)'. The '# seasons' field is set to 4. The 'Basis for smoothing' is set to 'Centered Moving Average'. The 'Seasonal Factor Scaling' is set to 'Rescale: set average to 1'. The data table below shows the following values:

Period	Demand(y)
Past period 2	125
Past period 3	150
Past period 4	141
Past period 5	116
Past period 6	134
Past period 7	159
Past period 8	152
Past period 9	123
Past period 10	142
Past period 11	168
Past period 12	165

Annotations in the image provide the following instructions:

- Specify the number of seasons (4 for quarterly data)
- Select Multiplicative Decomposition from the drop-down menu.
- Input the data
- Specify Centered Moving Average approach
- Select Rescale: set average to 1

Using Software

PROGRAM 5.6B – QM for Windows Output

Method: Multiplicative Decomposition (seasonal) # seasons (use 4 for qtrs, 12 for yrs): 4

Forecasting Results

The final forecast is obtained by multiplying the trend (unadjusted) forecast by the seasonal indices (factors).

The seasonal indices (factors) are shown here.

The trend line is shown here over two lines.

Measure			Unadjusted Forecast	Seasonal Factor	Adjusted Forecast
Error Measures		13	155.25	0.849	131.81
Bias (Mean Error)	0.001	14	157.594	0.963	151.687
MAD (Mean Absolute Deviation)	0.905	15	159.937	1.131	180.959
MSE (Mean Squared Error)	1.7	16	162.281	1.057	171.535
Standard Error (denom=n-2-4=6)	1.844	17	164.625	0.849	139.769
MAPE (Mean Absolute Percent Error)	0.595%	18	166.968	0.963	160.71
Regression line (unadjusted forecast)		19	169.312	1.131	191.566
Demand(y) = 124.784		20	171.655	1.057	181.444
+ 2.344 * time		21	173.999	0.849	147.728
Statistics		22	176.342	0.963	169.733
Correlation coefficient	0.8	23	178.686	1.131	202.172
Coefficient of determination	0.65	24	181.03	1.057	191.353
		25	183.373	0.849	155.686
		26	185.717	0.963	178.756

Using Regression with Trend and Seasonal

- Multiple regression can be used to forecast both trend and seasonal components
 - One independent variable is time
 - Dummy independent variables are used to represent the seasons
 - An additive decomposition model

$$\hat{Y} = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$$

where

X_1 = time period

X_2 = 1 if quarter 2, 0 otherwise

X_3 = 1 if quarter 3, 0 otherwise

X_4 = 1 if quarter 4, 0 otherwise

Using Regression with Trend and Seasonal

PROGRAM 5.7A – Excel QM Multiple Regression Initialization

The screenshot shows the 'Spreadsheet Initialization' dialog box with the following fields and values:

- Title: Turner Industries
- Sheet name: (empty)
- Number of (past) periods of data: 12
- Name for period: Period
- (Use A for A, B, C ... or a for a, b, c ...)
- Number of independent variables(X): 4
- Name for x's: X

Two callout boxes provide instructions:

- Enter the number of past period of data.
- Enter the number of independent variables.

Buttons at the bottom: Help, Cancel, OK.

Using Regression with Trend and Seasonal

PROGRAM 5.7B –
Excel QM Multiple
Regression Output

	A	B	C	D	E	F
1	Turner Industries					
2						
3	Forecasting		Mu			
4	Enter the data in the shaded area.			Enter the values of Y and X1-X4 as shown.		
5						
6						
7	Data					
8		Y	X 1	X 2	X 3	X 4
9	Period 1	108	1	0	0	0
10	Period 2	125	2	1	0	0
11	Period 3	150	3	0	1	0
12	Period 4	141				
13	Period 5	116				
14	Period 6	134				
15	Period 7	159				
16	Period 8	152	8	0	0	1
17	Period 9	193	9	0	0	0
18	Period 10	187	10	0	1	0
19	Period 11	168	11	0	0	1
20	Period 12	143	12	0	1	0
21						
22	Coefficients	104.104	2.313	15.688	38.708	30.063
23						
24						
25	Forecast	134.167	13	0	0	0

The regression coefficients are shown here.

Enter the values of the variables to obtain any forecast.

Using Regression with Trend and Seasonal

- Regression equation

$$\hat{Y} = 104.1 + 2.3X_1 + 15.7X_2 + 38.7X_3 + 30.1X_4$$

- Forecasts for first two quarters next year

$$\hat{Y} = 104.1 + 2.3(13) + 15.7(0) + 38.7(0) + 30.1(0) = 134$$

$$\hat{Y} = 104.1 + 2.3(14) + 15.7(1) + 38.7(0) + 30.1(0) = 152$$

Using Regression with Trend and Seasonal

- Regression
 - Different from the results using the multiplicative decomposition method
 - Use MAD or MSE to determine the best model

$$\hat{Y} = 104$$

- Forecasts for first two quarters next year

$$\hat{Y} = 104.1 + 2.3(13) + 15.7(0) + 38.7(0) + 30.1(0) = 134$$

$$\hat{Y} = 104.1 + 2.3(14) + 15.7(1) + 38.7(0) + 30.1(0) = 152$$

Monitoring and Controlling Forecasts

- **Tracking signal** measures how well a forecast predicts actual values
 - **Running sum of forecast errors (RSFE)** divided by the MAD

$$\begin{aligned}\text{Tracking signal} &= \frac{\text{RSFE}}{\text{MAD}} \\ &= \frac{\hat{a}(\text{forecast error})}{\text{MAD}}\end{aligned}$$

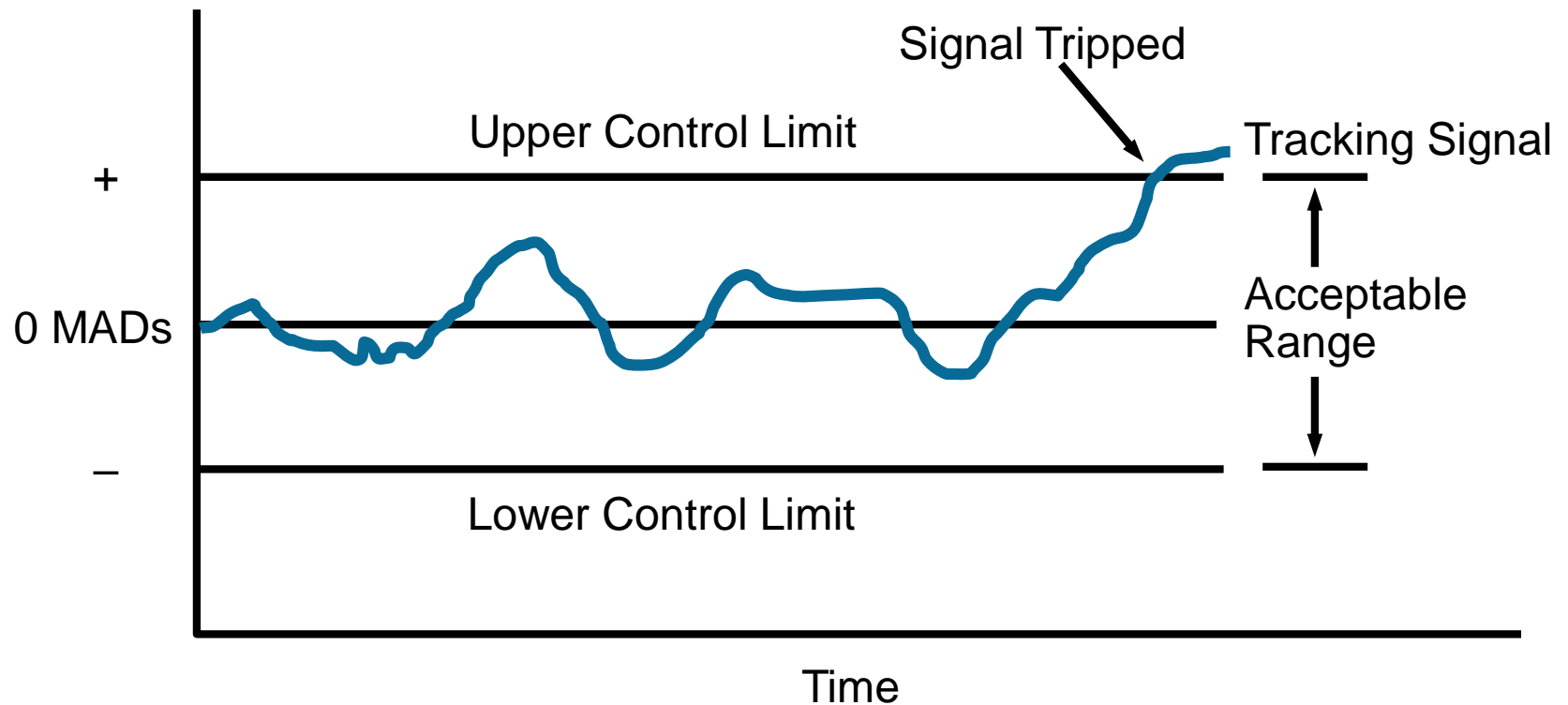
$$\text{MAD} = \frac{\hat{a}|\text{forecast error}|}{n}$$

Monitoring and Controlling Forecasts

- Positive tracking signals indicate demand is greater than forecast
- Negative tracking signals indicate demand is less than forecast
- A good forecast will have about as much positive error as negative error
- Problems are indicated when the signal trips either the upper or lower predetermined limits
- Choose reasonable values for the limits

Monitoring and Controlling Forecasts

FIGURE 5.7 – Plot of Tracking Signals



Kimball's Bakery Example

- Quarterly sales of croissants (in thousands)

TIME PERIOD	FORECAST DEMAND	ACTUAL DEMAND	ERROR	RSFE	FORECAST ERROR	CUMULATIVE ERROR	MAD	TRACKING SIGNAL
1	100	90	-10	-10	10	10	10.0	-1
2	100	95	-5	-15	5	15	7.5	-2
3	100	115	+15	0	15	30	10.0	0
4	110	100	-10	-10	10	40	10.0	-1
5	110	125	+15	+5	15	55	11.0	+0.5
6	110	140	+30	+35	35	85	14.2	+2.5

For Period 6:

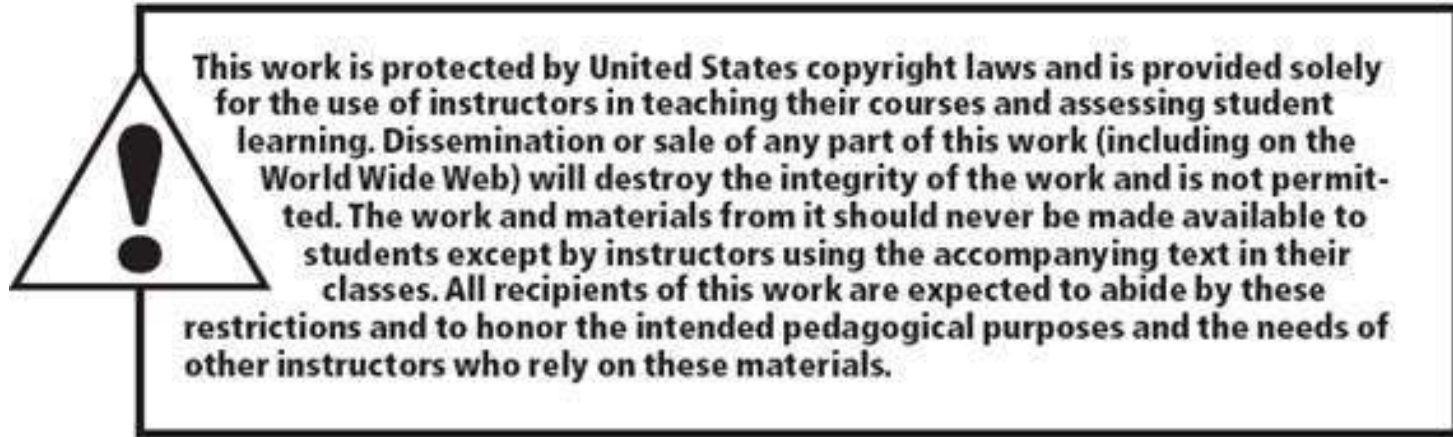
$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{85}{6} = 14.2$$

$$\text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}} = \frac{35}{14.2} = 2.5 \text{ MADs}$$

Adaptive Smoothing

- Computer monitoring of tracking signals and self-adjustment if a limit is tripped
- In exponential smoothing, the values of α and β are adjusted when the computer detects an excessive amount of variation

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