



**FINE 6140 (Financial Microeconomics)**

**Take-Home Mid-Term Examination**

**Due Date: 6/8/2016**

**Modigliani-Miller Irrelevance Propositions**

The Modigliani-Miller theorem proposes that in a perfect market, capital structure is irrelevant. Proposition 1 states that the value of an unlevered firm is identical to the value of a similar firm with leverage in its capital structure. In circumstances where there is a tax advantage to debt, the value of the levered firm will exceed its unlevered counterpart by the amount of the debt tax shield.

Proposition 1:

$$V^{LEVERED} = V^{UNLEVERED}$$

$$V^{LEVERED} = V^{UNLEVERED} + T_c D$$

$T_c$  is the corporate tax rate and  $D$  is the face value of debt. Proposition 2 states that the required equity return on an investment partially financed through leverage rises in proportion to the debt to equity ratio. However, given the tax advantage to debt, the weighted average cost of capital (WACC) actually declines as more leverage is used. In fact, in a perfect market, the lowest cost alternative would be to finance all projects with 100% debt.

Proposition 2:

$$i_E^{LEVERED} = i_E^{UNLEVERED} + \frac{D}{E} (i_E^{UNLEVERED} - i_D)$$

$$i_E^{LEVERED} = i_E^{UNLEVERED} + \frac{D}{E} (i_E^{UNLEVERED} - i_D) (1 - T_c)$$

1. Consider two firms with identical assets. Company A is all equity financed and company B is financed with a mix of debt and equity. There are also two equally likely states of the world which will occur at time 1. Company B has debt with a face value of \$45,000 that is due at time 1.

Market Value of Assets (T=1)	Company A	Company B
Good State	\$120,000	\$120,000
Bad State	\$30,000	\$30,000

Demonstrate that under the assumptions of the Modigliani-Miller theorem,  $V^A = V^B$ .

2. Assume the same information from the previous question. Now, suppose Company A's equity instrument trades for \$78,750, whilst company's B's equity instrument trades for \$37,500 and its debt instrument trades for \$37,500. In the absence of market frictions, does this represent an arbitrage opportunity? If it does, how would you take advantage of this opportunity?
3. As my good friend Aimee can tell you, short selling in the real world is [not straightforward](#). Assume the same information from the previous question. If you answered the question correctly, you should have engaged in a short transaction on one of the firm's equity instruments. Now, suppose that your broker has an initial margin requirement of 50%, and a maintenance margin of 30%. At what price would you receive a margin call?
4. Assume a Modigliani-Miller world, with a tax advantage to debt, in which Company C is unlevered and would like to include debt in its capital structure. The firm is presently valued at \$35,000 with 5,000 shares outstanding. The plan is to issue \$10,000 of new perpetual debt (that will be used to repurchase old stock) with an interest rate of 8%. The corporate tax rate is 25%. Calculate the value of Company C if they make this change.
5. Assume a Modigliani-Miller world in which Company D is currently financed entirely with equity, but would like to include debt in its capital structure. Also, assume that there are three states of the world that may occur at time 1 (good, fair and bad). The current and proposed capital structures are listed below, and the simplified income statements for each state of the world are listed in the table after:

Variable	Current	Proposed
Assets	\$20,000	\$20,000
Debt	\$0	\$8,000
Equity	\$20,000	\$12,000
Debt/Equity Ratio	0.00%	66.67%
Yield	N/A	8.00%
Shares Outstanding	400	240
Share Price	\$50	\$50

Current	Good	Fair	Bad
EBIT	\$1,000	\$2,000	\$3,000
Interest	\$0	\$0	\$0
EBT	\$1,000	\$2,000	\$3,000

Fill out the table below that calculates [operating ROA](#). Also, calculate “operating ROE” and [EPS](#) on an EBT basis for Company D under the new capital structure i.e. these two ratios use EBT as opposed to Net Income in the numerator.

Proposed	Good	Fair	Bad
EBIT	\$1,000	\$2,000	\$3,000
Interest			
EBT			
Operating ROA			
Operating ROE			
EPS			

6. Assume the same information about Company D from the previous question. Also, assume that you have perfect information about which states of the world CAN\* occur, but not which one will actually occur, and that all three states are equally likely. Calculate the standard deviation of the operating ROE under both capital structures. Explain your results. \*Because we are accounting for all possibilities, please use the [population version](#) of the standard deviation equation (Scroll down to the population variance chapter on Wiki).
7. Assume the same information from the previous problem about Company D. Why is Company D’s operating ROA the same irrespective of its capital structure?
8. As my good friend Aimee can tell you, there are a variety of financial products one can use to mitigate the consequences of negative outcomes. This process is called hedging (Read the document on the last page of the test). Assume the same information from the previous problem. Show how one could hedge the exposure to the levered version of Company D, so as to replicate the payoffs of its unlevered state. For example, you might be an investor who does not like the decision to change the capital structure. Assume you have \$2,000 to work with. [Slide 25](#) of this PowerPoint presentation may be helpful.
9. The pecking order theory is an apparent violation of Modigliani-Miller. Explain, in one page, what the pecking order theory is and how it more closely approximates real-life than the MM world.
10. Consider firm E. Firm E has a state (S) dependent value  $V(S)$ . Firm E also has a state dependent loss function  $L(S)$  that occurs in the event of catastrophe (the building burns down, the federal government bans its products due to safety concerns, the firm hires my other good friend Amy, as opposed to my good friend, Aimee etc...) Now, also suppose that the firm can be restored to full capacity after the catastrophe with a state dependent investment  $I(S)$ . Show how under debt and equity financing, there exist states of the world in which the firm will not be restored, even though restoration is a positive NPV proposition. For full credit, plot a graph that shows the relationship between dollars in state S (vertical axis) against the states of the world (increasing values on the horizontal axis indicate better states).

## a) Arbitrage

M&M argued that the external investors could engage in arbitrage to create an increased return on shares that were underleveraged. They could increase risk but raise the expected rate of return by employing debt to buy the shares.

The counter argument is that this is not a perfect offsetting tool. There are additional transaction costs, and external debt is more dangerous to maintain under varying returns and asset values, and more dangerous in terms of legal liability to the external or individual investor than the internal debt of the firm.

## b) Hedging

External investors could reduce the risk on a firm which was over-leveraged by purchasing both the shares and the bonds (debt) of the firm. This might reduce the expected return but it would provide a minimum fail safe position (i.e., the final value of the bonds) in case of failure.

The counter argument involves the problem of transaction costs and the fact that the offset in holding the bonds is far from perfect in case of failure and bankruptcy. The legal and transaction costs of going through bankruptcy and re-organization is likely to erode a large part of the value of the firm.

The major contribution of the M&M model is to show that each type of financing, debt or equity, effects the cost of the other. Nevertheless when the costs of failure, bankruptcy, re-organization, and various transaction costs are considered, it is clear that the tradeoff is not likely to be perfect. Perhaps we can sum up by noting that the central difficulty of the pure M&M model is the problem of the asymmetry of information. Should not a judicious financial management knowing the environmental conditions of their firm do a better job of setting up the financial structure than the outside investor? And if this is so, have we not arrived at the notion of an optimal capital structure?

Finally, the empirical evidence that financial structures are not random, but appear to be significantly different for varying classes of firms' points in the direction of the existence of optimal capital structure.

Nevertheless, M&M made an important contribution. It is clear that the bounds of the optimal capital structure are much broader than theory would suggest. Secondly, the M&M hypothesis sharpens the argument and more clearly points out the tax advantage (the tax deductibility of interest) of debt under our current corporation income tax laws.

M&M recognized the cost of capital implications of interest deductibility in their original 1958 study. M&M held that the interest

deductibility feature of corporate taxation leads to a decreasing cost of capital as the debt ratio rises. By 1963, M&M formulated the before-tax earnings yield, the ratio of expected earnings before interest and taxes,  $\bar{X}$ , to the market value of the firm,  $\hat{v}$ , as:

$$\frac{\bar{X}}{\hat{v}} = \frac{\rho^A}{1-t} \left[ 1 - t \frac{D}{V} \right]$$

Due to the tax advantage the cost of capital of the firm decreases with leverage, and the value of the firm will rise with the use of debt.

## 7. THE OPTIMAL CAPITAL STRUCTURE AND THE M&M HYPOTHESIS

The difference between optimal capital structure theory and the M&M hypothesis can be exaggerated. Both models emphasize the point that the use of one class of financing has rebound effects on the costs of the rest of the financial structure. In the optimal model, the overall cost of capital at any given time is constant within the range of the optimal capital structure. Debt or equity financing or some combination may be used for any particular project, as long as the financial mix is kept within an optimal range. Nevertheless, because every type of financing has interactions with the other sources of financing, the return on a project is not to be compared to the direct cost of its mode of financing but to the overall cost of the financial mix.

In the M&M model, the interaction between different types of financing is complete so there is no optimal financial structure. Thus the firm's overall cost of capital at any point of time is constant at the proper financial mix, or it is constant regardless of the mix. Most importantly, both of these views are in opposition to the sequential cost models, in which the cost of capital depends on the financing which is being used currently, so that the cost is lowest when the firm uses retained earnings, rises for outside borrowing, and becomes still higher when borrowing capacity is strained and additional funds depend on the flotation of new shares. In short, in making real investment decisions, Schwartz and M&M agree that the appropriate discount variable is not the immediate financial source but the overall cost of capital.<sup>10</sup>