1. Use the echelon method to solve the given system of two equations in two unknowns. Check your answers.

   $$16x - 3y = 10$$
   $$2x + 6y = 14$$

   Select the correct choice below and fill in any answer boxes in your choice.

   ○ **A.** The solution of the system is _____ .
   (Simplify your answer. Type an ordered pair.)

   ○ **B.** There are infinitely many solutions. The solution is $\left(\rule{2cm}{0.4pt} , y \right)$, where y is any real number.

   ○ **C.** There is no solution.

2. For the following system of equations in echelon form, tell how many solutions there are in nonnegative integers.

   $$4x + 2y + 4z = 100$$
   $$y - 4z = 30$$

   Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   ○ **A.** There are _____ nonnegative solutions.

   ○ **B.** There are infinitely many solutions.

   ○ **C.** There is no solution.

3. A person has $18,000 invested in stock A and stock B. Stock A currently sells for $50 a share and stock B sells for $80 a share. If stock B doubles in value and stock A goes up 50%, his stock will be worth $31,000. How many shares of each stock does he own?

   The person owns _____ shares of stock A and _____ shares of stock B.

4. A bank teller has a total of 80 bills in five-, ten-, and twenty-dollar denominations. The total value of the money is $88

   (a) Find the total number of solutions.
   (b) Find the solution with the smallest number of five-dollar bills.
   (c) Find the solution with the largest number of five-dollar bills.

   (a) Let x be the number of five-dollar bills, y be the number of ten-dollar bills, and z be the number of twenty dollar bills. Translate the given information into two equations, using x, y, and z as the variables.

   _____ = 80

   _____ = 880

   (Do not factor. Use integers or decimals for any numbers in the expressions.)

   There are a total of _____ solutions.

   (b) The solution with the smallest number of five-dollar bills is when the bank teller has _____ five(s), _____ tens, and _____ twenties.

   (c) The solution with the largest number of five-dollar bills is when the bank teller has _____ five(s), _____ tens, and _____ twenties.

5. Write an augmented matrix for the following system of equations.

   $2x - 5y + 3z = -7$
   $4x - 9y + 5z = 4$
   $ 4y - 9z = -6$

   The entries in the matrix are $\begin{bmatrix} \underline{} & \underline{} & \underline{} & | & \underline{} \\ \underline{} & \underline{} & \underline{} & | & \underline{} \\ \underline{} & \underline{} & \underline{} & | & \underline{} \end{bmatrix}$.

6. Replace R_2 by $(-8)R_1 + R_2$.

   $\begin{bmatrix} 1 & 7 & -1 & | & -9 \\ 8 & 6 & 9 & | & 1 \\ -6 & 4 & 1 & | & 1 \end{bmatrix}$ $(-8)R_1 + R_2$

   $(-8)R_1 + R_2 = \begin{bmatrix} \underline{} & \underline{} & \underline{} & | & \underline{} \\ \underline{} & \underline{} & \underline{} & | & \underline{} \\ \underline{} & \underline{} & \underline{} & | & \underline{} \end{bmatrix}$

7. Use the Gauss-Jordan method to solve the system of equations.

$$y = -2 + x$$
$$y = -1 + z$$
$$z = 1 - x$$

Select the correct choice below and fill in any answer boxes within your choice.

○ **A.** There is one solution. The solution is $\left(\underline{\hspace{2cm}} , \underline{\hspace{2cm}} , \underline{\hspace{2cm}} \right)$, in the order x, y, z.
(Type an exact answer in simplified form.)

○ **B.** There are infinitely many solutions. The solution is $\left(\underline{\hspace{2cm}} , \underline{\hspace{2cm}} , z \right)$, where z is any real number.
(Type an exact answer in simplified form.)

○ **C.** There is no solution.

8. A company is upgrading office technology by purchasing inkjet printers, LCD monitors, and additional memory chips. The total number of pieces of hardware purchased is 45. The cost of each inkjet printer is $100, the cost of each LCD monitor is $128, and the cost of each memory chip is $85. The total amount of money spent on new hardware came to $4476. They purchased two times as many memory chips as they did LCD monitors. Determine the number of each that was purchased.

Write a linear system of equations. Let x be the number of inkjet printers, y be the number of LCD monitors, and z be the number of memory chips. Choose the correct answer below.

○ **A.** $100x + 85z = 4476$
$x + y + z = 45$
$-2y + z = 0$

○ **B.** $x + 45y + 85z = 4476$
$x + y + z = 128$
$-2y + z = 100$

○ **C.** $100x + 128y + 85z = 4476$
$x + y + z = 45$
$-2y + z = 0$

○ **D.** $100x + 128y + 85z = 4476$
$x + y + 2z = 45$
$-2y + z = 33$

The company purchased \underline{\hspace{2cm}} inkjet printers, \underline{\hspace{2cm}} LCD monitors, and \underline{\hspace{2cm}} memory chips.
(Simplify your answers.)

9. Find the size of the matrix. Determine if it is a square, column, or row matrix.

$$\begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

The matrix is \underline{\hspace{2cm}} $\times$ \underline{\hspace{2cm}} .

Choose the correct type of matrix.

○ no special type
○ square matrix
○ column matrix
○ row matrix

10. Find the values of the variables in the equation.

$$\begin{bmatrix} s-8 & t+4 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ -3 & r \end{bmatrix}$$

r = _____

s = _____

t = _____

11. Add the following matrices.

$$\begin{bmatrix} 5 & 9 & 6 \\ -9 & -9 & -9 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 9 \\ -1 & -8 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ A. $\begin{bmatrix} 5 & 9 & 6 \\ -9 & -9 & -9 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 9 \\ -1 & -8 \end{bmatrix} =$ _____

○ B. The matrices cannot be added.

12. A dietician prepares a diet specifying the amounts a patient should eat of the four basic food groups: group I, meats; group II, fruits and vegetables; group III, breads and starches; group IV, milk products. Amounts are given in "exchanges" that represent 1 oz (meat), 1/2 cup (fruits and vegetables), 1 slice (bread), 8 oz (milk), or other suitable measurements. Complete parts (a) through (c) below.

(a) The number of "exchanges" for breakfast for each of the four food groups, respectively, are 1, 2, 1, and 1; for lunch, 2, 3, 2, and 1; and for dinner, 3, 2, 3, and 1. Write a 3×4 matrix using this information.

_____ (Type an integer or decimal for each matrix element.)

(b) The amounts of fat, carbohydrates, and protein (in appropriate units) in each food group, respectively, are as follows. Use this information to write a 4×3 matrix.

    Fat: 6, 0, 0, 8
    Carbohydrates: 0, 12, 14, 12
    Protein: 6, 2, 3, 5

_____ (Type an integer or decimal for each matrix element.)

(c) There are 8 calories per exchange of fat, 3 calories per exchange of carbohydrates, and 9 calories per exchange of protein. Summarize this data in a 3×1 matrix.

_____ (Type an integer or decimal for each matrix element.)

13. Let A = $\begin{bmatrix} 7 & -2 \\ -3 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & 0 \\ -3 & -5 \end{bmatrix}$.

Find $-4A + 4B$.

$-4A + 4B =$ _____

14. The sizes of two matrices A and B are given. Find the sizes of the product AB and the product BA, whenever these products exist.

    A is 5×4, and B is 4×4.

    Find the size of the product AB. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

    ○ A. The size of product AB is _____ × _____ .

    ○ B. The product AB does not exist.

    Find the size of the product BA. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

    ○ A. The size of product BA is _____ × _____ .

    ○ B. The product BA does not exist.

15. Find the following matrix product, if possible.

$$\begin{bmatrix} 7 & 9 & -4 \\ -3 & 7 & -1 \end{bmatrix} \begin{bmatrix} -9 \\ -3 \\ 7 \end{bmatrix}$$

    Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

    ○ A. $\begin{bmatrix} 7 & 9 & -4 \\ -3 & 7 & -1 \end{bmatrix} \begin{bmatrix} -9 \\ -3 \\ 7 \end{bmatrix} =$ _____

    ○ B. The product is not possible.

16. If A is a square matrix then $A^2 = AA$. Let $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$. Find A^2.

    $A^2 =$ _____ (Simplify your answer.)

17.

Four departments of a company need to order the following amounts of the same products.

Department	Paper	Tape	Binders	Memo Pads	Pens
1	11	3	4	5	7
2	7	3	2	3	8
3	5	7	1	0	10
4	0	2	4	4	5

The unit price (in dollars) of each product is given below for two suppliers.

	Supplier A	Supplier B
Paper	4	5
Tape	1	1
Binders	4	3
Memo Pads	3	3
Pens	1	2

(a) Use matrix multiplication to get a matrix showing the comparative costs for each department for the products from the two suppliers.

$$
\begin{array}{cc}
& \quad A \qquad\qquad B \\
\begin{array}{c} \text{Dept. 1} \\ \text{Dept. 2} \\ \text{Dept. 3} \\ \text{Dept. 4} \end{array} &
\left[\begin{array}{cc} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{array} \right]
\end{array}
$$

(b) Find the total cost over all departments to buy products from each supplier. From which supplier should the company make the purchase?

○ **A.** The total cost for supplier A is $ \underline{\qquad} and the total cost for supplier B is $ \underline{\qquad} , so the company should make the purchase from supplier B.

○ **B.** The total cost for supplier A is $ \underline{\qquad} and the total cost for supplier B is $ \underline{\qquad} , so the company should make the purchase from supplier A.

18. Find the inverse, if it exists, for the given matrix.

$$
\begin{bmatrix} 3 & 2 \\ 3 & 3 \end{bmatrix}
$$

Find the inverse. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** The inverse is \underline{\qquad} . (Type a matrix, using an integer or simplified fraction for each matrix element. Do not factor out a scalar multiple.)

○ **B.** There is no inverse of the given matrix.

19. Find the inverse, if it exists, for the given matrix.

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & -1 & 0 \\ 3 & -1 & 7 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** The inverse matrix is _____ . (Type a matrix, using an integer or simplified fraction for each matrix element. Do not factor out a scalar multiple.)

○ **B.** There is no inverse of the given matrix.

20. Find the production matrix for the following input-output and demand matrices using the open model.

$$A = \begin{bmatrix} 0.1 & 0.3 \\ 0.45 & 0.1 \end{bmatrix} \qquad D = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

The production matrix is $\begin{bmatrix} \underline{\hphantom{xxxxx}} \\ \underline{\hphantom{xxxxx}} \end{bmatrix}$.

(Round the final answer to the nearest hundredth as needed. Round all intermediate values to four decimal places as needed.)

21. An economy depends on two basic products, wheat and oil. To produce 1 metric ton of wheat requires 0.25 metric tons of wheat and 0.31 metric tons of oil. Production of 1 metric ton of oil consumes 0.09 metric tons of wheat and 0.13 metric tons of oil. Find the production that will satisfy a demand for 490 metric tons of wheat and 940 metric

tons of oil. The input-output matrix is $A = \begin{bmatrix} 0.25 & 0.09 \\ 0.31 & 0.13 \end{bmatrix}$.

_____ metric tons of wheat is required to satisfy the demand.
(Round the final answer to the nearest whole number as needed. Round the elements of the inverse matrix to two decimal places as needed.)

_____ metric tons of oil is required to satisfy the demand.
(Round the final answer to the nearest whole number as needed. Round the elements of the inverse matrix to two decimal places as needed.)