Advanced Topics **LIE ALGEBRAS** **(Date Due: 30^{th}July)**

ASSIGNMENT 2

Question 1.
Let $\mathfrak{L}$ be the real vector space $\mathbb{R}^3$. Given $x, y \in \mathfrak{L}$, define

$$[x, y] := x \times y,$$

where $\times$ denotes the usual *cross product* of vectors.

Sow that $\mathfrak{L}$ is a Lie algebra and determine its structure constants relative to the standard basis for $\mathbb{R}^3$.

Question 2.
Let δ be a derivation of the Lie algebra $\mathfrak{L}$. Show that if δ commutes with every inner derivation, then

$$\delta(\mathfrak{L}) \subseteq \mathcal{C}(\mathfrak{L}),$$

where $\mathcal{C}(\mathfrak{L})$ denotes the *centre* of $\mathfrak{L}$.

Question 3.
Let $x \in \mathfrak{gl}(n, \mathbb{F})$ have n distinct eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ in $\mathbb{F}$. Prove that the eigenvalues of ad_x are the n^2 scalars

$$\lambda_i - \lambda_j, \qquad (1 \leq i,\ j \leq n).$$

(Note that only $n^2 - n + 1$ scalars are paiwise distinct from each other since $\lambda_i - \lambda_i = 0$ for all i.)