3. Now let's consider solutions to the wave equation with only one boundary, at $x = 0$.

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2}(x,t) = c^2 \dfrac{\partial^2 u}{\partial x^2}(x,t), & (x, t > 0) \\ u(0,t) = 0, & (t > 0) \\ u(x,0) = f(x), & (x > 0) \\ \dfrac{\partial u}{\partial t}(x,0) = g(x), & (x > 0) \end{cases}$$

Start with the function $v(x,t) = \frac{\partial u}{\partial t}(x,t) + c\frac{\partial u}{\partial x}(x,t)$.

What PDE does $v(x,t)$ solve?

Show that $v(x,t) = g(x+ct) + cf'(x+ct)$ for all $x, t \geq 0$.

4 and 5. Now solve the first-order PDE

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) + c\dfrac{\partial u}{\partial x}(x,t) = v(x,t), & (x, t > 0) \\ u(0,t) = 0, & (t > 0) \\ u(x,0) = f(x), & (x > 0) \end{cases}$$

Note: The solution formula is different depending on whether $x > ct$ or $x < ct$. Treat these two cases separately.