

DELHI SCHOOL OF ECONOMICS  
 Course 001: Microeconomic Theory  
 Problem Set 2

1. Show that if  $\succsim$  is transitive, then  $\succ$  and  $\sim$  are also transitive.
2. Assume  $\succsim$  is transitive. Show that if  $\mathbf{x}^1 \sim \mathbf{x}^2$  and  $\mathbf{x}^2 \succ \mathbf{x}^3$ , then  $\mathbf{x}^1 \succ \mathbf{x}^3$ .
3. Consider the following (lexicographic) preferences:

$$(x_1, x_2) \succ (y_1, y_2) \text{ if } x_1 > y_1, \text{ or if } x_1 = y_1 \text{ and } x_2 > y_2$$

Can these preferences be represented by a utility function? Which of our four axioms do these preferences violate?

4. A consumer's preferences over two goods are described by the following utility function:

$$U(x_1, x_2) = (x_1 - \alpha_1)^{\beta_1} (x_2 - \alpha_2)^{\beta_2}$$

where  $\alpha_i$  and  $\beta_i > 0$  ( $i = 1, 2$ ) are constants.

- (a) Consider the utility function:

$$V(x_1, x_2) = \gamma \log(x_1 - \alpha_1) + (1 - \gamma) \log(x_2 - \alpha_2)$$

For what value of  $\gamma$  does the function  $V(x_1, x_2)$  represent the same preferences as  $U(x_1, x_2)$ ?

- (b) Derive the Marshallian demand functions for the two goods when preferences are represented by  $V(x_1, x_2)$ , assuming the value of  $\gamma$  derived above.
5. Derive the Marshallian and Hicksian demand functions as well as expenditure and indirect utility functions for the following preferences:

- (a)  $u(\mathbf{x}) = \prod_{i=1}^n x_i^{\alpha_i}$  where  $\alpha_i > 0$  and  $\sum_{i=1}^n \alpha_i = 1$ .

- (b)  $u(x_1, x_2) = \max\{ax_1, ax_2\} + \min\{x_1, x_2\}$

- (c)  $u(x_1, x_2) = x_1^2 + x_2^2$

- (d)  $u(x_1, x_2) = ax_1 + bx_2$

- (e)  $u(x_1, x_2) = x_1x_2 + x_1 + x_2$

6. Prove that for additive utility functions of the form  $U = u_1(x_1) + u_2(x_2) + \dots + u_n(x_n)$  with  $u'_i > 0$  and  $u''_i < 0$ , none of the goods can be Giffen goods.
7. An agent derives utility from consuming food ( $F$ ) and leisure ( $L$ ). Her preferences are captured by the utility function  $u(F, L) = L^{\frac{1}{3}} F^{\frac{2}{3}}$ . She has a total endowment of time given by  $T$  hours, which she must divide between leisure and work. The hourly wage rate is  $w$ . In addition to any wage income, the agent also earns income  $m$  from other sources. The agent spends all her income to buy food, which can be purchased at price  $p$ . Derive her demand function for food and supply function of labour. Under what condition will labour supply be 0?
8. An agent's preferences are captured by the utility function:  $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$ . The agent owns 5 units of good 1 and 3 units of good 2. Refer to these quantities as his endowments. He can earn some money by selling some of his endowments and can buy additional quantities of either good with the money thus earned. The market prices of the two goods are  $p_1$  and  $p_2$ .
  - (a) Set up the agent's maximization problem and solve for the optimal choices of  $x_1$  and  $x_2$ .
  - (b) For what range of prices is this agent a net buyer of good 1 and net seller of good 2?

9. The government of Banana Republic spends its budget ( $I$ ) on two divisible goods, schools ( $X$ ) and missiles ( $Y$ ). The government's utility function is

$$u(x, y) = xy^2$$

- (a) Derive the Marshallian demand functions for missiles and schools, and the government's indirect utility function.
  - (b) Assume  $I = 120$ ,  $p_x = 20$  and  $p_y = 40$ . How many missiles and schools will the government buy?
  - (c) An aid agency, Bleeding Heart Inc., offers aid to the government of Banana Republic in the form of a subsidy for schools. Specifically, Bleeding Heart promises to pay half the price of every school (and part thereof) that Banana Republic buys. Find the new consumption bundle of Banana Republic, and calculate the *total* amount of money Bleeding Heart will end up spending on the programme.
  - (d) An economist, Mr. Level Head, argues that the aid agency could have enabled Banana Republic reach the same level of utility (as in part (c)), but at a lower cost to itself, by simply giving them a cash grant, and allowing the government to spend it any way it wanted. Calculate the amount of cash grant that will be necessary, and state whether the economist's claim is true.
  - (e) The director of Bleeding Heart retorts that her organization's objective is not to make the Banana Republic government happier, but to increase their purchase of schools. If the aid agency were to give aid in the form of a cash award, calculate the amount they would need to give so that the recipient government bought as many schools as in part (c). Do your calculations support the aid agency's counter-argument?
10. A consumer's preferences are captured by the utility function

$$u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \quad (0 < \alpha < 1)$$

where  $x_1$  is the amount (in Rupees) she spends on her own needs and  $x_2$  is the amount she donates to charity. The consumer's (before-tax) income  $y$  is subject to a proportional income tax at rate  $t$  ( $0 < t < 1$ ).

- (a) Derive the optimal choice of own consumption and charitable donations as functions of  $t$  and other parameters of the model. How does a tax cut (reduction in  $t$ ) affect the size of charitable donations?
- (b) Now suppose charitable donations are tax deductible, i.e., for every rupee given to charity, the individual can claim a refund of Rs  $t$  from the Income tax department, which she can spend on her own needs. Once again, set up the optimization problem and derive the choices of  $x_1$  and  $x_2$  as functions of  $t$  and other parameters. What is the effect of a tax cut (reduction in  $t$ ) on charitable donations in this scenario?
- (c) Give an interpretation of the difference in results between parts (a) and (b) in terms of income and substitution effects.
- (d) Are the qualitative results in parts (a) and (b) (i.e., effect of tax cuts on charity) sensitive to some special property of the assumed Cobb-Douglas form? In general, if we only know that preferences satisfy the usual properties (monotonicity and convexity), can we predict whether charity contributions will rise or fall in response to tax cuts in either case?