Paper Code: 715189
Algebra and Discrete Mathematics
Lecturer: Ji Ruan, Kate Lee
Assignment 1
Due 4:00pm, Friday, 15 August 2014

Name .... **ID number.........................**

Question	**Marks Possible**	**Marks Given**
1	8	
2	12	
3	10	
4	10	
5	9	
6	10	
7	16	
Total	75	

Instructions:

Please attach this sheet to the front of your assignment.

The assignment must be handed in before 4.00 p.m. on **Friday 15 August 2014**. For the students in AUT South Campus, you need to submit the assignment answers to Kate Lee. For the students in City Campus, you need to put the assignment answers in the assignment box which is in the reception area for the School of Computer and Mathematical Sciences. The address for WT building is 2-14 Wakefield St on level 1.

This is an individual assignment. The point of the assignment is for you to go through the process of discovery for yourself. Copying someone else's work will not achieve this. Plagiarism has occurred where a person effectively and without acknowledgement presents as their own work the work of others. That may include published material, such as books, newspapers, lecture notes or handouts, material from the www or other students' written work. That work also includes computer output.

The School of Computing and Mathematical Sciences regards any act of cheating including plagiarism, unauthorised collaboration and theft of another student's work most seriously. Any such act will result in a mark of zero being given for this part of the assessment and may lead to disciplinary action.

Please sign to signify that you understand what this means, and that the assignment is your own work.

Signature: ....

Question 1 (8 marks) *Using Euclid's algorithm, determine if the following rational numbers are in reduced form. If not, write own their reduced forms. Show your working.*

1. $\frac{179}{78}$
2. $\frac{288}{468}$

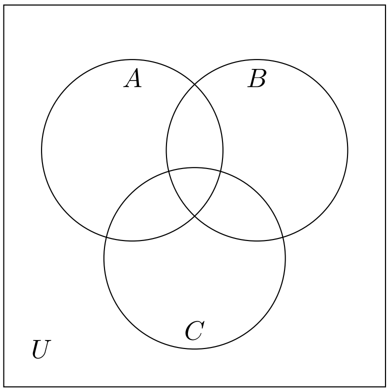
Question 2 (12 marks) *Argue why the following statements hold.*

1. $n(2n+1)(4n+1) \equiv 0 \ (mod\ 3)$
2. $\sqrt{3}$ *is not a rational number.*

Question 3 (10 marks) *Let* $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6\}$. *Show the following sets by enumeration:*

1. $\{x \mid x \notin A, x \in B \text{ and } x|25\}$
2. $\mathcal{P}(A - B)$
3. $\{X \mid X \subseteq A \text{ and } |X| = 2\}$
4. $\{X \mid X \subseteq A \text{ and } 1 \in X\}$
5. $\{X \mid X \subseteq A \text{ and } X \cap B \neq \emptyset\}$

Question 4 (10 marks) *Let* U, A, B, C *be as indicated in the diagram below:*



We've shown a proof of the first distributive law $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ *in Lecture 4. Here is the second distributive law:*

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

1. *Shade the area of* $A \cup (B \cap C)$ *in the venn diagram.*
2. *Give a proof using the similar arguments in Lecture 4.*

Question 5 (9 marks) *Let* $A = \{0, 1, \{2, 3, 4\}\}$*,* $B = \emptyset$*,* $C = \{1, 5\}$*,* $D = C \times \mathbb{N}$*. Which of the following are true and which are false?*

- $|A \cup C| = |A| + |C| - |A \cap C|$.
- $A \cap C = C - \{5, 2\}$.
- $\mathbb{N} \subseteq D$.
- $A \cap C - C = \emptyset$.
- $A \cap C = C$.

What are the cardinalities of the following sets?

- $\mathcal{P}(\mathcal{P}(B))$.
- $A \times C$.
- $A \times D$.
- C^5.

Question 6 (10 marks) *Let* A, B *be sets. Show the following is true*

$$\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A \cup B).$$

Question 7 (16 marks) *For each of the following binary relations, determine whether they are* reflexive, symmetric, antisymmetric *or* transitive*. Show your arguments.*

- $R_1 = \{(x, y) \in P^2 \mid x \text{ is an ancestor of } y\}$*, where* P *is a set of people.*
- $R_2 = \{(X, Y) \in \mathcal{P}(A)^2 \mid X \subseteq Y \text{ and } X \neq Y\}$*, where* A *is a non-empty set.*