PURE MATHEMATICS 212

Multivariable Calculus

ASSIGNMENT 2

1. [2 marks] Name and sketch the surface: $z = 4x^2 + y^2 + 8x - 2y$

2. [4 marks] (a) Find a parametric equation of the curve of intersection of the paraboloid $z = 3 - x^2 - y^2$ and the plane $z = 2y$.
(b) Find an equation of the orthogonal projection of this curve to the xy-plane.

3. [4 marks] The curves below are given by their vector equations. Describe them in Cartesian coordinates. What are their geometric names?

(a) $\mathbf{r}=(3\sin e^t)\,\mathbf{i} + (3\cos e^t)\,\mathbf{j}$. (b) $\mathbf{r} = -2\,\mathbf{i} + t\,\mathbf{j} + (t^2 - 1)\,\mathbf{k}$.

4. [3 marks] Define the notion of a smooth curve. For which values of the parameter t is the curve

$$\mathbf{r} = t^3\cos(t)\mathbf{i} + \sin(t^2)\mathbf{j} + t^2\mathbf{k}$$

smooth? Justify your answer.

5. [3 marks] Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be differentiable vector-valued functions of t. Prove that

$$\frac{d}{dt}[\mathbf{u}\cdot[\mathbf{v}\times\mathbf{w})] = \frac{d\mathbf{u}}{dt}\cdot[\mathbf{v}\times\mathbf{w}] + \mathbf{u}\cdot[\frac{d\mathbf{v}}{dt}\times\mathbf{w}] + \mathbf{u}\cdot[\mathbf{v}\times\frac{d\mathbf{w}}{dt}]$$

Hint: Apply the product laws for dot and cross product.

6. [4 marks] (a) Evaluate $\int[(te^t)\,\mathbf{i} + \ln t\,\mathbf{j}]\,dt$;

(b) Find the arc length of the curve $\mathbf{r}(t) = (3\cos t)\mathbf{i} + (3\sin t)\mathbf{j} + 4t\mathbf{k};\ 0 \le t \le 2\pi$.