Matrix Multiplication

#1 For each pair of matrices given below, report the size of each matrix (an $m \times n$ matrix has m rows and n columns). Determine the size of their product and multiply them, if possible. If the matrix multiplication is not possible, use their sizes to explain why not.

(a)

$$\begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \\ 0 & 2 & 1 \\ 4 & 3 & -2 \\ -5 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 5 & -2 \\ 0 & 8 & -3 & 7 \\ 6 & -3 & 0 & 5 \end{pmatrix} = $$

↑   ↑   ↑

<u>5</u> × <u>3</u>   __ × __   __ × __

(b)

$$\begin{pmatrix} 2 & 4 & -1 \\ -4 & -5 & 3 \\ 2 & -5 & -4 \\ -6 & 0 & 7 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -6 & 0 \\ 8 & -1 \end{pmatrix} = $$

↑   ↑   ↑
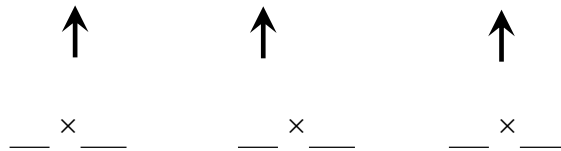
__ × __   __ × __   __ × __

#1 (continued)

(c) $(0.2 \quad 0.3 \quad -0.1) \begin{pmatrix} 40 \\ -60 \\ 80 \end{pmatrix} = \begin{pmatrix} \end{pmatrix}$

↑   ↑   ↑

__×__   __×__   __×__
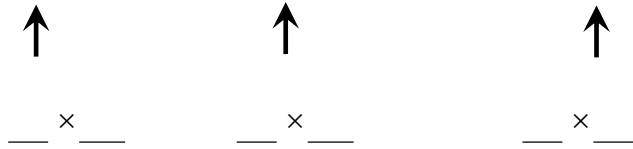
(d) $\begin{pmatrix} -6 \\ 3 \\ 0 \end{pmatrix} (3 \quad -7 \quad 8) = \begin{pmatrix} \end{pmatrix}$

↑   ↑   ↑

__×__   __×__   __×__

(e) $\begin{pmatrix} 1 & 3 & 4 & 0 \\ -3 & -6 & -7 & 2 \\ 3 & 3 & 0 & -4 \end{pmatrix} \begin{pmatrix} 0.158 & 0.006 & 0.002 & 0.030 & 0.001 \\ 0.005 & 0.264 & 0.043 & 0.009 & 0.008 \\ 0.026 & 0.150 & 0.355 & 0.013 & 0.014 \\ 0.329 & 0.056 & 0.049 & 0.363 & 0.020 \\ 0.008 & 0.008 & 0.033 & 0.029 & 0.341 \end{pmatrix} = \begin{pmatrix} \end{pmatrix}$

↑   ↑   ↑

__×__   __×__   __×__
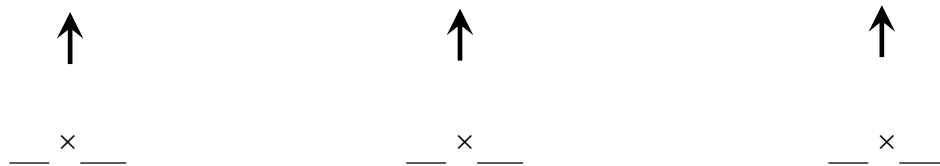
#2   (a)   When you have finished #1, have a teaching assistant check all your answers. If your assistant finds a mistake, s/he is not allowed to tell you the answer. Your assistant is only allowed to tell you which answer is wrong and can remind you how to solve the problem.

(b)   Use Maple to check each of your answers in #1. Print out your results. Include both the commands used as well as the output. What happens in Maple, when the multiplication is not possible?

#3   Compute the matrix-vector product $A\vec{v}$. A matrix times a column gives a   _?_

(a)
$$\begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \\ 0 & 2 & 1 \\ 4 & 3 & -2 \\ -5 & 0 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 7 \\ 5 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$$

Compare your answer with #1 (a). What do you observe? Be specific.

↑   ↑   ↑

__ × __   __ × __   __ × __

#4   Compute the vector-matrix product $\vec{w}A$. A row times a matrix gives a   _?_

(a) $\begin{pmatrix} 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 5 & -2 \\ 0 & 8 & -3 & 7 \\ 6 & -3 & 0 & 5 \end{pmatrix} = \begin{pmatrix} & & & \end{pmatrix}$

Compare your answer with #1 (a). What do you observe? Be specific.

↑   ↑   ↑

__ × __   __ × __   __ × __

#5 For each problem below, compute the matrix-vector product, $A\vec{v}$. What pattern do you observe? Be specific.

(a) $\begin{pmatrix} 1 & 3 & 4 & 0 \\ -3 & -6 & -7 & 2 \\ 3 & 3 & 0 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 3 & 4 & 0 \\ -3 & -6 & -7 & 2 \\ 3 & 3 & 0 & -4 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 3 & 4 & 0 \\ -3 & -6 & -7 & 2 \\ 3 & 3 & 0 & -4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 3 & 4 & 0 \\ -3 & -6 & -7 & 2 \\ 3 & 3 & 0 & -4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$

#6 For each problem below, compute the vector-matrix product, $\vec{w}A$. What pattern do you observe? Be specific.

(a) $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 3 & 4 & 0 \\ -3 & -6 & -7 & 2 \\ 3 & 3 & 0 & -4 \end{pmatrix} = \begin{pmatrix} \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 3 & 4 & 0 \\ -3 & -6 & -7 & 2 \\ 3 & 3 & 0 & -4 \end{pmatrix} = \begin{pmatrix} \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 4 & 0 \\ -3 & -6 & -7 & 2 \\ 3 & 3 & 0 & -4 \end{pmatrix} = \begin{pmatrix} \end{pmatrix}$

#7 Look at the assignments plus solutions from previous years and practice more matrix multiplication problems. Make sure you are comfortable with matrix multiplication. If you have problems or questions, please let me know.