

# 1 PSYCHROMETICS

Psychrometry is the study of the characteristics of moist air. We will see soon that evaporation of moisture from the skin can have a significant impact on thermal comfort. The rate of evaporation depends on the amount of moisture in the surrounding air. Therefore, before we can give serious consideration to thermal comfort in buildings, we must understand the characteristics of moist air.

## Basic Definitions

There are several indices for defining the amount of moisture in air. For our purposes, we will find it useful to assume that moist air is a mixture of ideal gases – dry air and water vapor. This assumption generally gives errors of less than 1% compared to the true thermodynamic properties.

### Partial Pressure, $p_w$

For an ideal gas mixture, Dalton's law states that the sum of the partial pressures of the individual components is equal to the total pressure.

$$p = p_w + p_{da} \quad (1-1)$$

The total pressure is typically driven by atmospheric pressure. At altitude, the atmospheric pressure can be estimated from the elevation,  $H$ , in feet above sea level.

$$p_{atm} = p_{sealevel} e^{-H/27,000} \quad (1-2)$$

### Relative Humidity, $\phi$

Moist air at a given temperature can only hold so much moisture before it becomes saturated. (The temperature of moist air is often referred to as the dry-bulb temperature, which is the temperature we all know from experience.) Referring to basic thermodynamics, if you increase the pressure of pure water vapor at a fixed temperature, you will eventually reach the saturation pressure and the water vapor will condense to a liquid. In a mixture of ideal gases, moist air is saturated at a given temperature when the partial pressure of the water vapor reaches that saturation pressure. The relative humidity is simply the ratio of the partial pressure of the water vapor in the moist air mixture to the saturation pressure of water vapor at the same temperature.

$$\phi = \frac{p_w}{p_{sat}} \quad (1-3)$$

When saturated, moist air has a relative humidity of 1.0 or 100%.

### **Humidity Ratio, $W$**

Humidity ratio of moist air is defined as the ratio of the mass of water vapor to the mass of dry air in the mixture. (Note, it is **not** the ratio of the mass of water vapor to the total mixture mass.) An ideal gas analysis can show that the humidity ratio is directly related to the partial pressures.

$$W = 0.622 \frac{P_w}{P - P_w} \quad (1-4)$$

The denominator is simply the partial pressure of the dry air and the coefficient of 0.622 is the ratio of the molecular weight of water vapor to that of dry air.

### **Dew Point Temperature, $T_{dew}$**

The dew point temperature of moist air is the temperature at which the moisture would condense if the moist air were cooled at a constant pressure. We defined the relative humidity in terms of the saturation pressure at a given temperature. The humidity ratio is defined in terms of the saturation temperature at a given partial pressure. The dew point temperature is a function only of the partial pressure. Using the definition of the humidity ratio above, the dew point temperature can also be considered a function only of humidity ratio.

### **Wet-Bulb Temperature, $T_{wb}$**

The wet-bulb temperature is an odd variable. It is defined only because it is easy to measure and it is often used in HVAC applications as an intermediate variable for calculating humidity ratio, dew point temperature, or relative humidity. It is measured simply by wrapping a wetted wick around a conventional “dry-bulb” thermometer. Blowing moist air across the wick causes water to evaporate, which depresses the measure temperature below the dry-bulb temperature of the air. When the moist air is relatively dry, the depression can be very large. As the air becomes more humid, the depression becomes smaller until, at saturation (100% relative humidity), the water will not evaporate and the web-bulb thermometer measures the same as the dry-bulb thermometer.

### **Enthalpy, $h$**

The enthalpy of a mixture of water vapor and dry air is the sum of the component enthalpies. For our purposes, we will find it convenient to express the enthalpy of moist air in terms of the energy per unit mass of dry air in the mixture. That is, enthalpy,  $h$ , will have units of Btu/lb<sub>da</sub> or kJ/kg<sub>da</sub>.

$$h = h_{da} + Wh_w \quad (1-5)$$

The enthalpies of dry air and water vapor are functions of temperature. Approximately,

$$\begin{aligned} h_{da} &= c_{pa}(T - T_{ref}) \\ h_w &= h_{g,ref} + c_{pw}(T - T_{ref}) \end{aligned} \quad (1-6)$$

$h_{g,ref}$  is the enthalpy of saturated water vapor at  $T_{ref}$ . In US (or IP) units,  $T_{ref} = 0^\circ\text{F}$  while in SI units,  $T_{ref} = 0^\circ\text{C}$ . With  $T_{ref} = 0$  for both sets of units, the equations can be combined as

$$h = c_{pa}T + W(h_{g,ref} + c_{pw}T) \quad (1-7)$$

Table 1–1: Values of Variables in Enthalpy Equation

Variable	US Units	SI Units
$c_{pa}$	0.240 Btu/lb <sub>da</sub> °F	1.0 kJ/kg <sub>da</sub> °C
$c_{pw}$	0.444 Btu/lb <sub>w</sub> °F	1.86 kJ/kg <sub>w</sub> °C
$h_{g,ref}$	1061.2 Btu/lb <sub>w</sub>	2501.3 kJ/kg <sub>w</sub>

### Specific Volume, $v$

Like the enthalpy, the specific volume is expressed as the volume of the mixture per unit mass of dry air. Since water vapor has a lower molecular weight than dry air, the specific volume of moist air is greater than that of dry air. (The density of moist air is less than that of dry air.)

### Psychrometric Chart

All of the above properties, with the exception of the partial pressure of water vapor, are conveniently given on a psychrometric chart. A psychrometric chart is constructed from the fundamental relationships among thermodynamic properties. A given chart is constructed assuming a specific atmospheric pressure and is comprised of a set of lines of constant property values. (Note that the atmospheric pressure appears in the relationship between humidity ratio and partial pressure.) With all these properties given on a single chart, it can be confusing. Figure 1-1 identifies the basic property values displayed on a typical psychrometric chart.

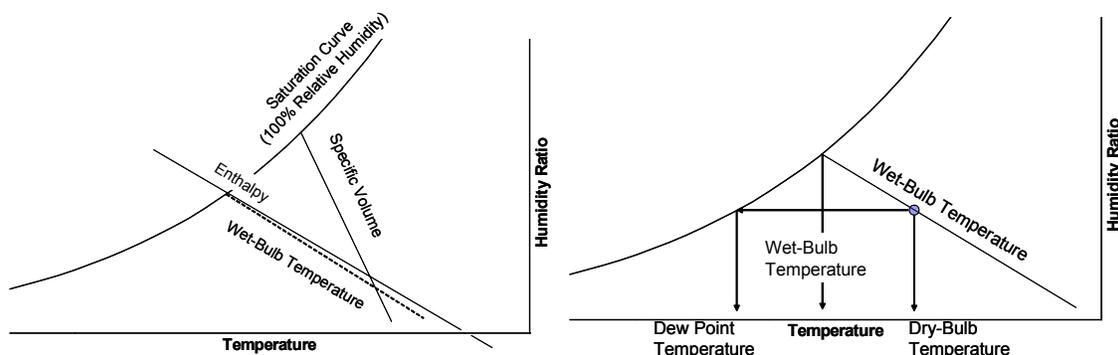


Figure 1-1 Basic Properties on Psychrometric Chart

The psychrometric chart has two main uses. First, it is useful to identify the properties of moist air. (If I know the temperature and relative humidity, I can find the enthalpy.) Second, it is very useful in analyzing processes in building systems. We will explore psychrometric processes later, but for now will focus on the psychrometric chart as a tool for property evaluation.

## Moisture in Buildings

Most discussions about the heating and cooling requirements of buildings focus on the need to control the temperature in the building to maintain comfort. If we don't control the temperature with heating or cooling equipment, the indoor temperature would be near the outdoor temperature. Actually it would typically be slightly above the outdoor temperature due to the influence of solar radiation and internal heat gains.

The same occurs with humidity. If we don't control the humidity in the building with humidifying or dehumidifying equipment, the indoor humidity ratio would be slightly above the outdoor humidity ratio due to the influence of internal moisture gains. Figure 1-2 shows hours of occurrence of outdoor conditions in Atlanta. The figure suggests that, in winter, indoor humidity will be approximately 0.001 – 0.006, which corresponds to an indoor relative humidity of approximately 5% – 35% at typical indoor temperatures. In summer, outdoor humidity is approximately 0.012 – 0.018, giving indoor relative humidity of 60% – 90%. These high indoor humidities cause two problems. First, they are inherently uncomfortable. Second, indoor dew-point temperatures are 62°F – 74°F. Any effort to cool air at this high dew-point temperature will surely result in condensation and corresponding dehumidification of the air. Therefore, any discussion of cooling also involves dehumidification, necessitating a need to discuss psychrometric process in HVAC systems.

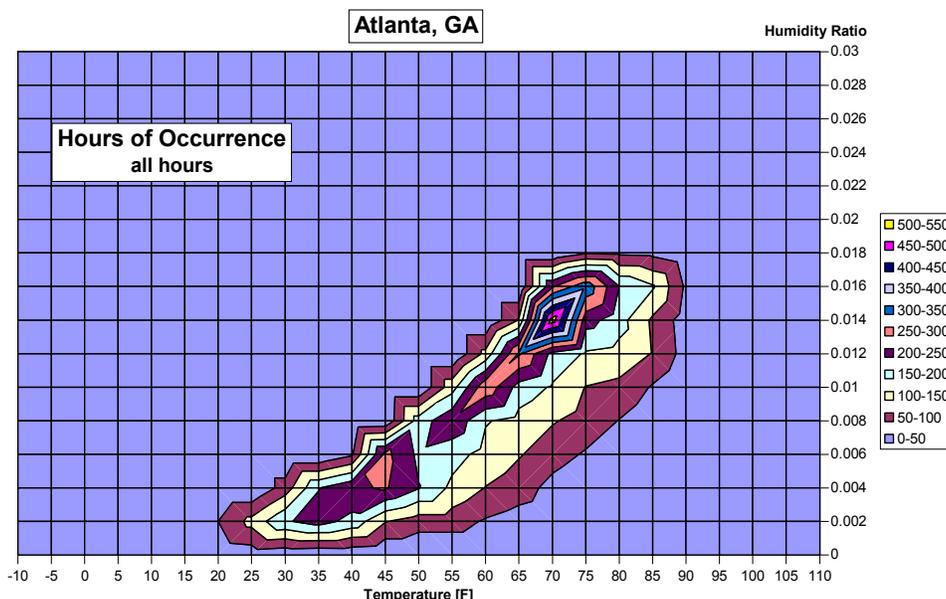


Figure 1-2 Outdoor air conditions in Atlanta

## Heating or Cooling of Moist Air, No Moisture Added or Removed

The humidity ratio of an air stream will remain constant if no moisture is added or removed. Figure 1-3 and Figure 1-4 show the equations and processes on a psychrometric chart.

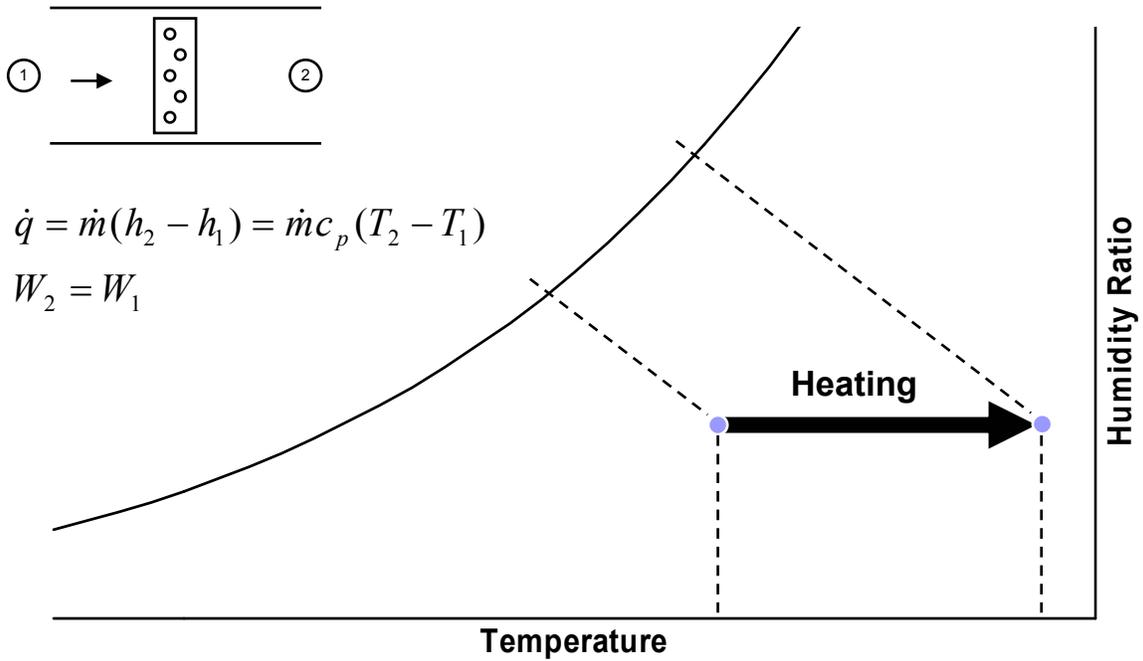


Figure 1-3 Heating of moist air, no moisture addition

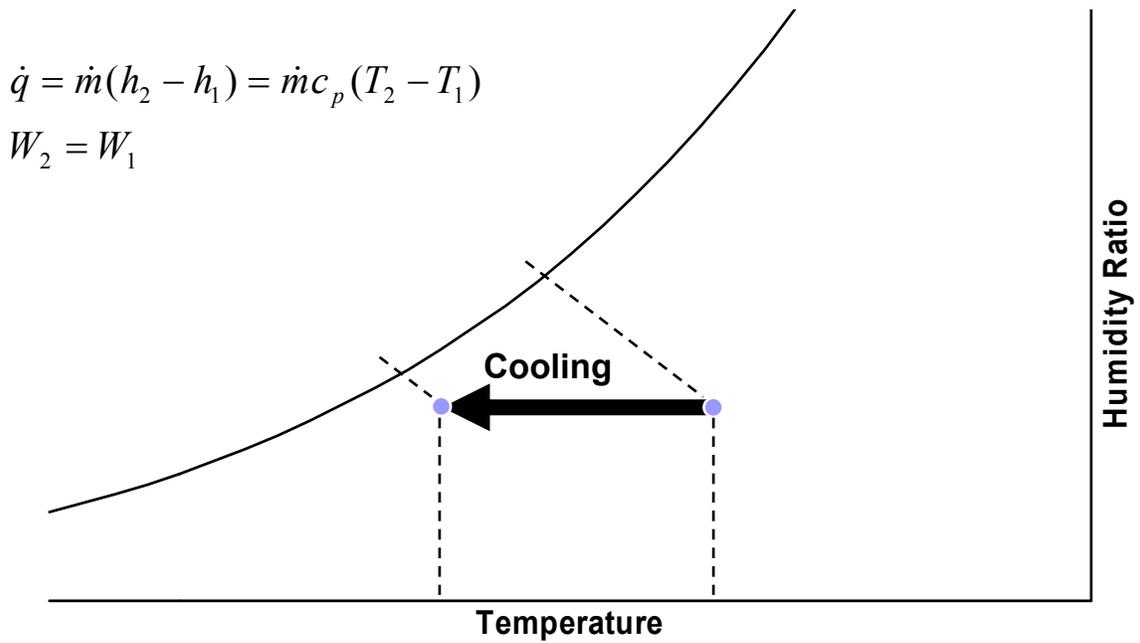


Figure 1-4 Cooling of moist air, no moisture removed

### Humidification

Air can be humidified by injecting water or steam into an air stream. If water or steam is injected at a mass flow rate  $m_w$  and an enthalpy  $h_w$ , simple energy and mass balances can be performed to

characterize the change in moist air enthalpy and humidity ratio. In these equations, it doesn't matter whether the moisture is added with liquid water or with steam. Note that liquid water has a much lower enthalpy than steam, due to the heat of vaporization,  $h_{fg}$ . Figure 1-5 shows the equations and process on a psychrometric chart. Notice that the enthalpy of the entering water defines the *slope* of the process line on the psychrometric chart.

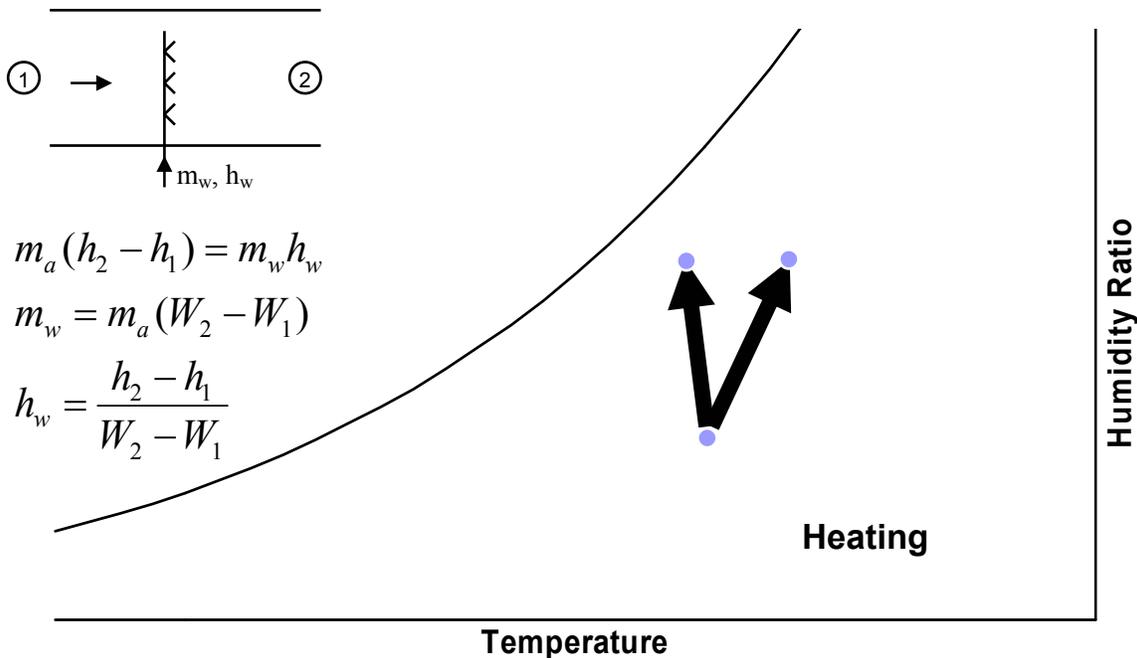


Figure 1-5 Humidification of moist air

### Cooling and Dehumidification

If moist air comes in contact with a surface that is below the dew-point temperature of the air, moisture will condense on the surface and the air will be dehumidified. Typical cooling coils in air conditioning systems operate at approximately 40° - 50°F, below the dew-point temperature of typical indoor air conditions. Mass and energy balances can be performed to relate the moist air properties to the energy and moisture removed from the air stream.

Figure 1-6 shows basic process between an entering and leaving air state. The figure also shows a third point, defined as the *apparatus dew point* (ADP). The ADP represents an effective coil surface condition and the apparatus dew point temperature can loosely be thought of as the coil temperature. The ADP is determined by extending the straight line between the entering and leaving air states until it reaches the saturation curve.

The fundamental mass and energy balances are given by the following.

$$\begin{aligned}
 m_w &= m_a(W_1 - W_2) \\
 m_a(h_1 - h_2) &= q + m_w h_w \approx q
 \end{aligned}
 \tag{1-8}$$

Since the enthalpy of the condensed water is typically small, it is common to relate the energy removed by the cooling coil,  $q$ , as the energy removed from the air. The total heat removed by the coil is commonly known as the total *capacity* of the coil. It is also common to separate the total energy removal into separate terms – one to account for the sensible cooling of the air and one to account for the energy required to dehumidify the air by condensing the water vapor. The total capacity is then the sum of the sensible and latent capacities.

$$\begin{aligned}
 q_{tot} &= m_a (h_1 - h_2) \\
 q_{sen} &= m_a c_p (T_1 - T_2) \\
 q_{lat} &= q_{tot} - q_{sen} = m_a h_{fg} (W_1 - W_2)
 \end{aligned}
 \tag{1-9}$$

Recall that the specific heat,  $c_p$ , account for the specific heats of both the air and the water vapor in the moist air mixture.

$$c_p = c_{p,a} + Wc_{p,v} \tag{1-10}$$

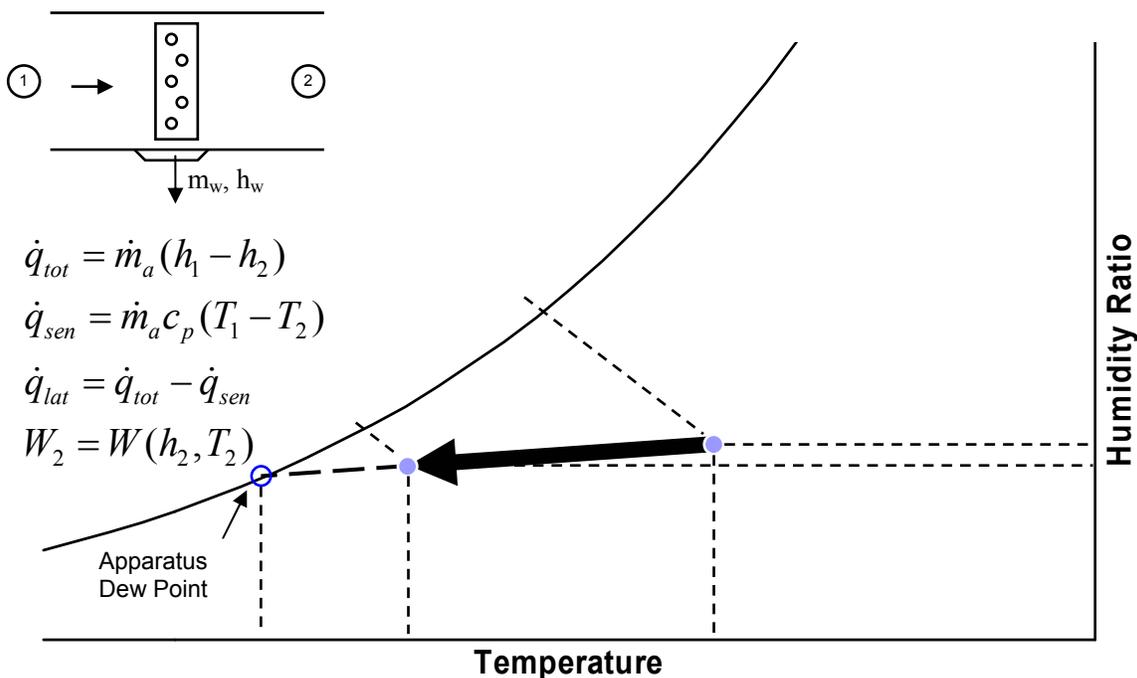


Figure 1-6 Cooling and dehumidifying moist air

Note that the specific heat is actually a function of the humidity ratio, which changes between point 1 and 2. Also, the latent heat of vaporization,  $h_{fg}$ , is a function of temperature. As a result, the sensible and latent capacities are rather loosely defined. Since the value of the specific heat is less variable than the heat of vaporization, it is common to calculate the specific heat from either the entering or leaving humidity ratio and then calculate the latent capacity as the difference between the total and sensible capacities.

The *sensible heat ratio* is defined as the ratio of the sensible capacity to the total capacity.

$$SHR = \frac{q_{sen}}{q_{tot}} = \frac{c_p(T_1 - T_2)}{h_1 - h_2} \quad (1-11)$$

The relationships among the psychrometric states, capacities, and sensible heat ratio are shown graphically in Figure 1-7. For a given pair of entering and leaving air states, the figure shows that sensible, latent, and total capacities are proportional to the differences in temperature, humidity ratio, and enthalpy, respectively. The SHR is defined by the *slope* of the line connecting the two points.

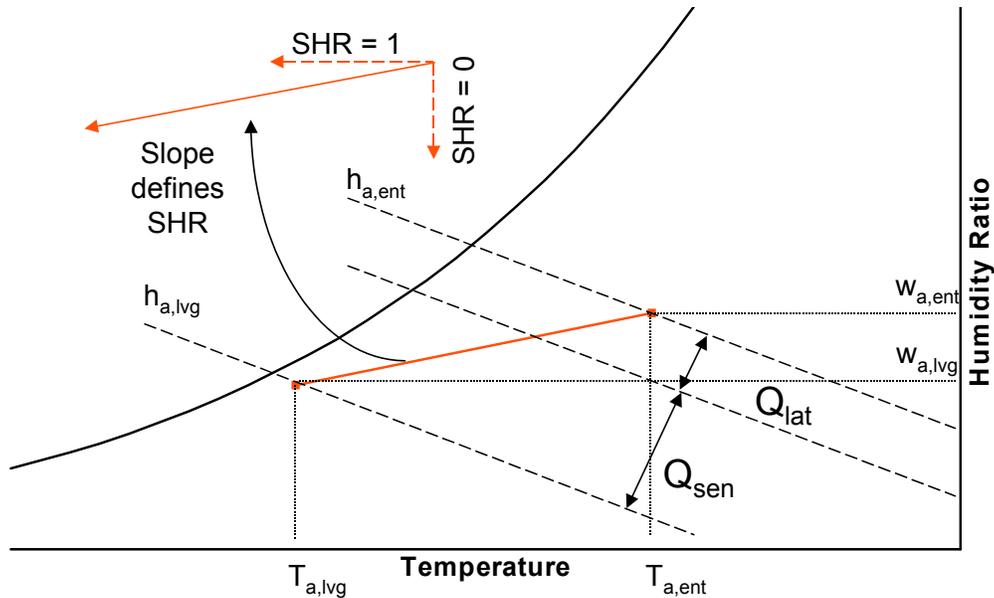


Figure 1-7 Coil capacities and sensible heat ratio on psychrometric chart

### Adiabatic Mixing

Air streams are regularly mixed in HVAC systems. The properties of the mixed moist air stream can be readily determined from those of the individual air streams. If streams 1 and 2 are mixed to form stream 3, a mass balance and energy balance allow calculation of the mixed air humidity ratio and enthalpy

$$\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2} \quad (1-12)$$

$$W_3 = \frac{\dot{m}_{a1}W_1 + \dot{m}_{a2}W_2}{\dot{m}_{a1} + \dot{m}_{a2}} \quad (1-13)$$

$$h_3 = \frac{\dot{m}_{a1}h_1 + \dot{m}_{a2}h_2}{\dot{m}_{a1} + \dot{m}_{a2}} \quad (1-14)$$

Intuitively, we also think of analogous temperature relationship in which the mixed air temperature is a mass weighted average of the entering temperatures. However, strictly speaking,

the relationship is only approximate, due to the effect of both temperature and moisture on the air enthalpy.

$$T_3 \approx \frac{\dot{m}_{a1}T_1 + \dot{m}_{a2}T_2}{\dot{m}_{a1} + \dot{m}_{a2}} \quad (1-15)$$

On a psychrometric chart, the mixed air state lies on a line connecting the two entering states. The process is shown in Figure 1-8

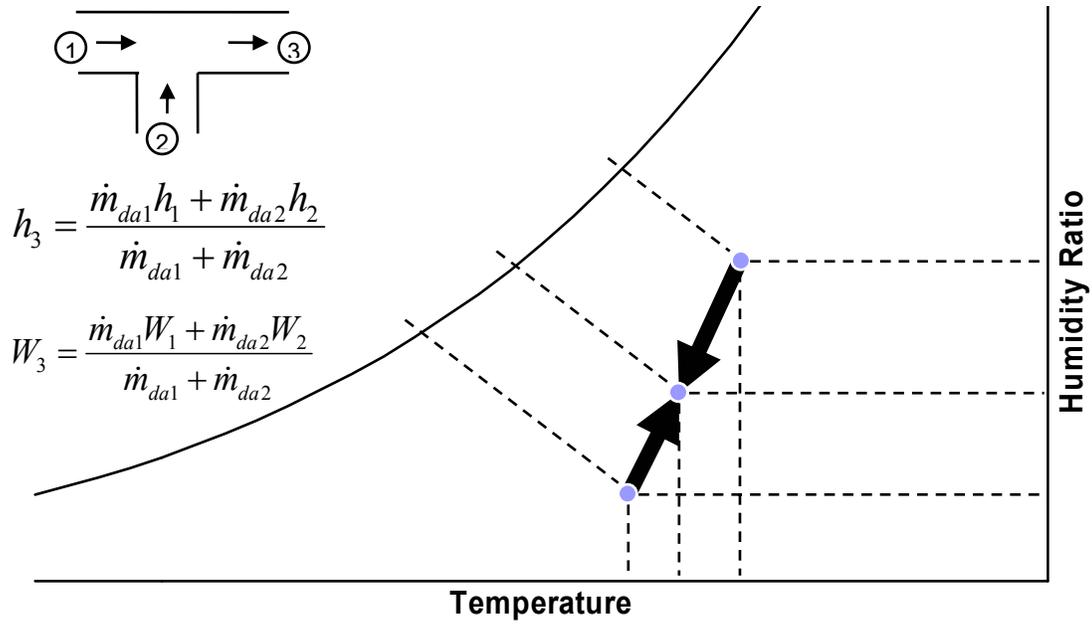


Figure 1-8 Adiabatic mixing of moist air

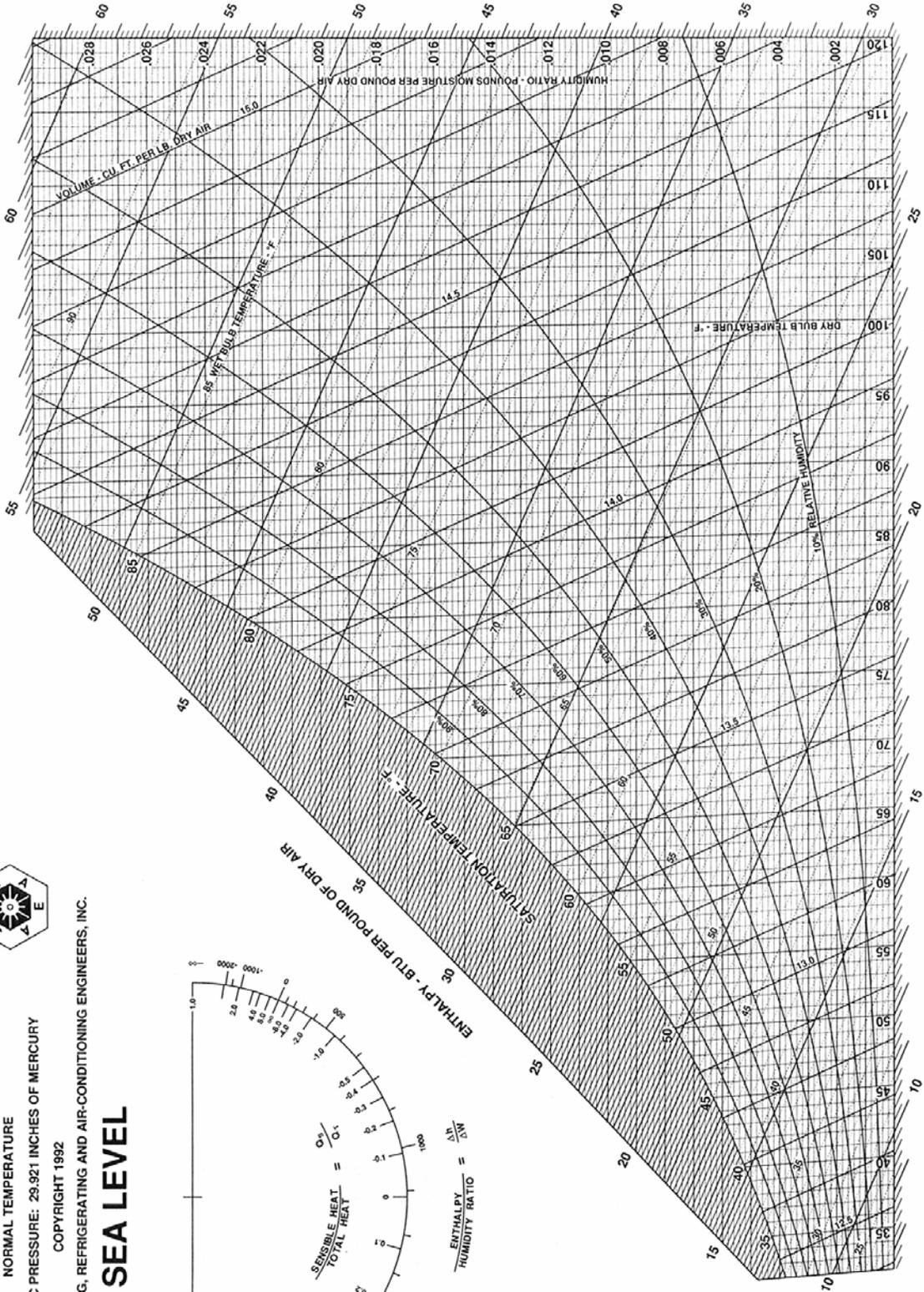
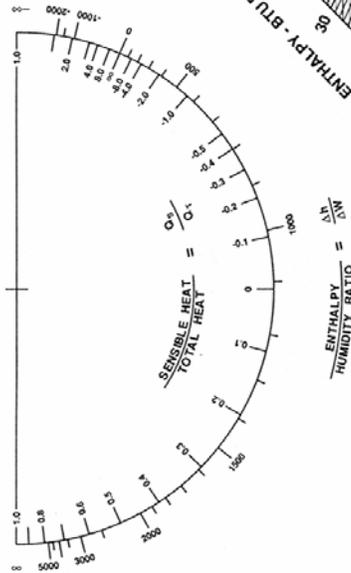
# Psychrometrics

## ASHRAE PSYCHROMETRIC CHART NO. 1

NORMAL TEMPERATURE  
BAROMETRIC PRESSURE: 29.921 INCHES OF MERCURY  
COPYRIGHT 1992

AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.

## SEA LEVEL



ENTHALPY - BTU PER POUND OF DRY AIR

Prepared by: CENTER FOR APPLIED THERMODYNAMIC STUDIES, University of Idaho

# Psychrometrics

## ASHRAE PSYCHROMETRIC CHART NO. 4

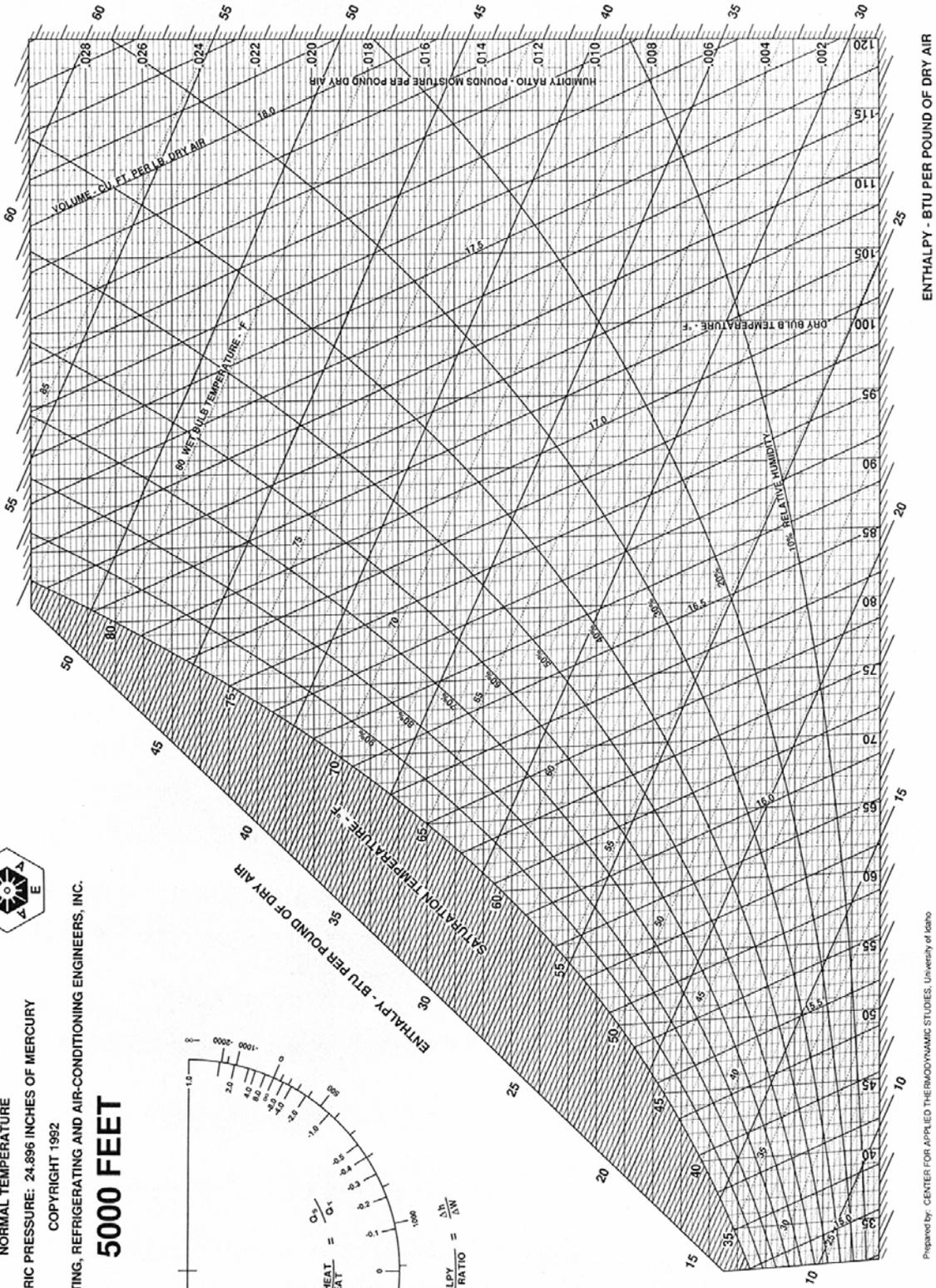
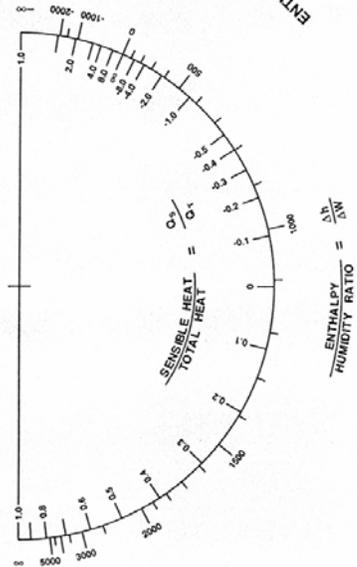
NORMAL TEMPERATURE

BAROMETRIC PRESSURE: 24.896 INCHES OF MERCURY

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**5000 FEET**



### Example Problems

Calculate the boiling point of water in Boulder.

Complete the following table for moist air at sea level.

$T_d$	$\phi$	W	h	$T_{wb}$	$T_{dew}$	v
95°F	30%					
80°F				67°F		

Complete the following table for moist air at an altitude of 5000 ft.

$T_d$	$\phi$	W	h	$T_{wb}$	$T_{dew}$	v
95°F	30%					
80°F				67°F		

Will water condense on a glass of ice water in this room today?

Complete the following table of property data for water

T, °F	p, psia	v, ft <sup>3</sup> /lb	Phase
80			Saturated vapor
	1		Saturated vapor
212			Saturated vapor
	12		Saturated vapor
80	0.1		

Use the Excel psychrometric spreadsheet on the website to solve the above problems.

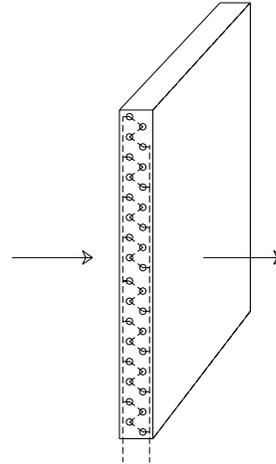
Air enters a cooling coil with a flow rate of 10,000 cfm at sea level. The air enters at a temperature of 80°F and 67°F web-bulb and leaves at 50°F and 90% relative humidity. What is the total cooling capacity of the coil? What is the sensible capacity of the coil? What is the latent capacity? What is the SHR of the coil? What is the apparatus dew-point temperature?

The central cubical area of an office building at sea level has a floor area of 4000 ft<sup>2</sup> and is occupied by 30 people who each produce 200 Btu/h of sensible heat and 250 Btu/h of latent heat. Heat gains from the lighting for the space produces a load of 1.2 W/ft<sup>2</sup>. (Heat gains from lights and people represent the only cooling load in the space.) The office is to be maintained at 72°F and 50% relative humidity. Conditioned air is supplied at 60°F to meet the sensible and latent loads. What is the load SHR? To meet the loads, what must be the mass flow rate and humidity ratio of the supply air?

### Psychrometric Process Problem #1

Moist air enters a heating coil at 40°F dry-bulb temperature and 36°F wet-bulb temperature at a rate of 2000 cfm. Barometric pressure is 14.696 psia. The air leaves the coil at a dry-bulb temperature of 140°F.

- a) How much heat is added to the air by the heating coil?

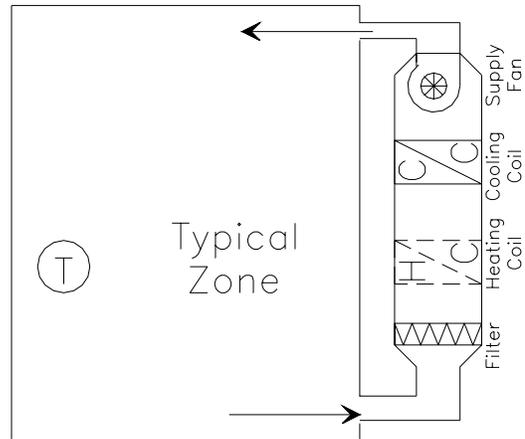


- b) If the heat is provided by hot water that enters the coil at 180°F and leaves at 160°F, what is the water flow rate?
- c) If the heat is provided by a steam coil with saturated steam at 250°F, what steam flow rate is required? Assume that the steam leaves the coil as a saturated liquid.

## Psychrometric Process Problem #2

A building space is to be maintained at 70°F dry-bulb and 30% RH when the barometric pressure is 14.696 psia. The total heating load on the space has been calculated to be 60,000 Btu/hr. Supply air is delivered to the space at a temperature of 120°F. The supply fan has a power consumption of 400 W.

- a) What is the volumetric flow rate of supply air required to meet the space heating load?

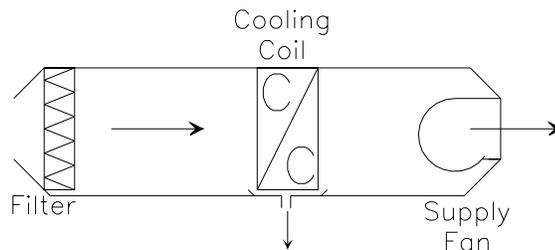


- b) What is the discharge air temperature leaving the coil?

### Psychrometric Process Problem #3

Moist air enters a cooling coil at 78°F dry-bulb temperature and 50% RH at a rate of 2400 cfm. Barometric pressure is 14.696 psia. The air leaves the coil at a dry-bulb temperature of 55°F and a dew-point temperature of 53°F. The condensate leaves at a temperature of 52°F. The supply fan has a power input of 1200 W.

- a) What is the total load (heat transfer rate) on the cooling coil?

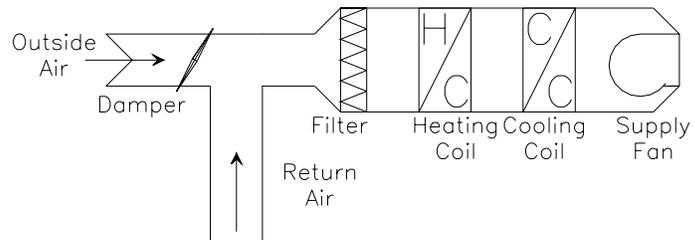


- b) What is the sensible heat factor for the coil?

- c) What are the supply air conditions?

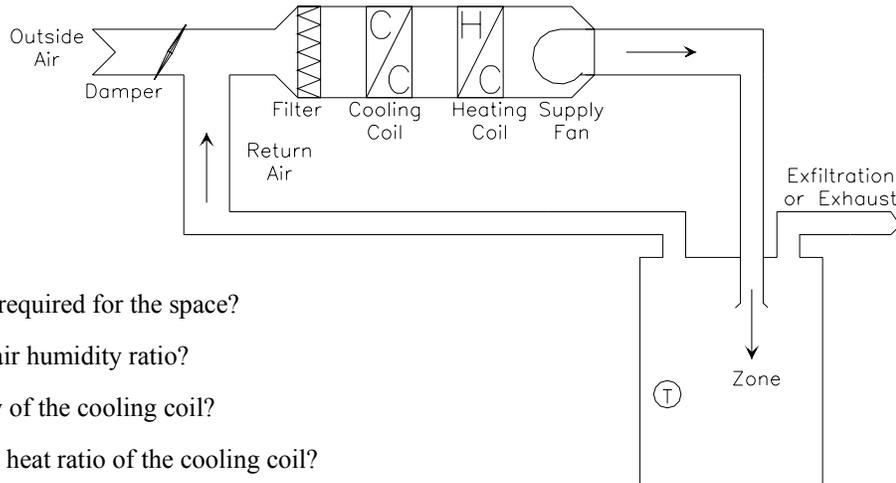
### Psychrometric Process Problem #4

Outdoor air is mixed with building return air in a mixing chamber prior to conditioning. The outdoor air is at 90°F and 40% RH with an airflow of 1000 cfm. The return air is 80°F and 67°F wet-bulb with an airflow rate of 3000 cfm. What is the mixed air condition?



### Psychrometric Process Problem #5

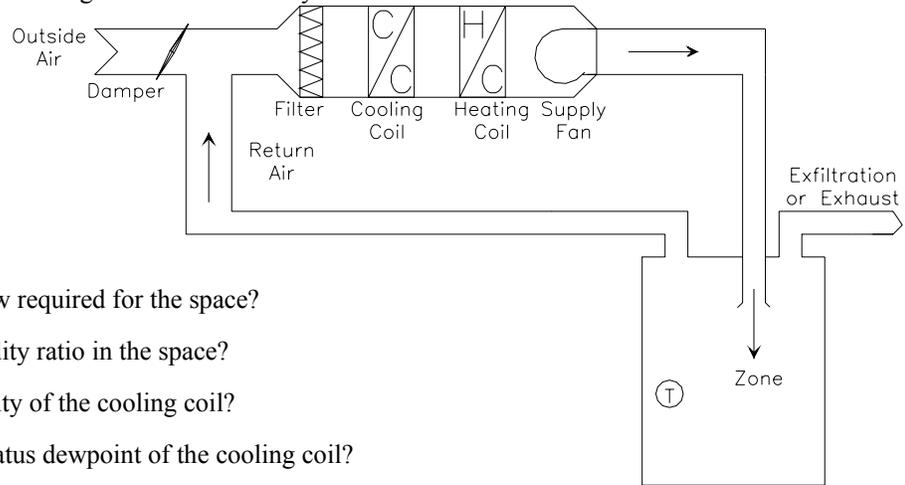
A building space is to be maintained at 78°F dry-bulb and 65°F wet-bulb. The total cooling load on the space has been calculated to be 60,000 Btu/hr of which 42,000 Btu/hr is sensible heat gain. Supply air is delivered to the space at a temperature of 58°F. For acceptable air quality, 500 cfm of outside air is required. Outdoor conditions are 90°F and 50% RH. A schematic diagram of the HVAC system is shown below.



- What is the airflow required for the space?
- What is the supply air humidity ratio?
- What is the capacity of the cooling coil?
- What is the sensible heat ratio of the cooling coil?

### Psychrometric Process Problem #6

A building space is to be maintained at 78°F dry-bulb. The total load on the space has been calculated to be 30,000 Btu/hr of which 27,000 Btu/hr is sensible heat gain. Supply air is delivered to the space at 58°F and 90% RH. The outdoor air mass flow rate is 25% of the total supply mass flow. Outdoor conditions are 80°F and 55% RH. A schematic diagram of the HVAC system is shown below.



- What is the airflow required for the space?
- What is the humidity ratio in the space?
- What is the capacity of the cooling coil?
- What is the apparatus dewpoint of the cooling coil?