EXPERIMENT THREE
DC CIRCUITS

EQUIPMENT NEEDED:

1) DC Power Supply
2) DMM
3) Resistors
4) ELVIS

THEORY

Kirchhoff's Laws:

Kirchhoff's Voltage Law: The algebraic sum of the voltages around any closed path is zero.

$$\sum_{i=1}^{N} v_i = 0 \qquad (3.1)$$

Kirchhoff's Current Law: The algebraic sum of the currents at any node is zero.

$$\sum_{i=1}^{N} i_i = 0 \qquad (3.2)$$

Series Circuits:

In a series circuit the current is the same through all the elements.

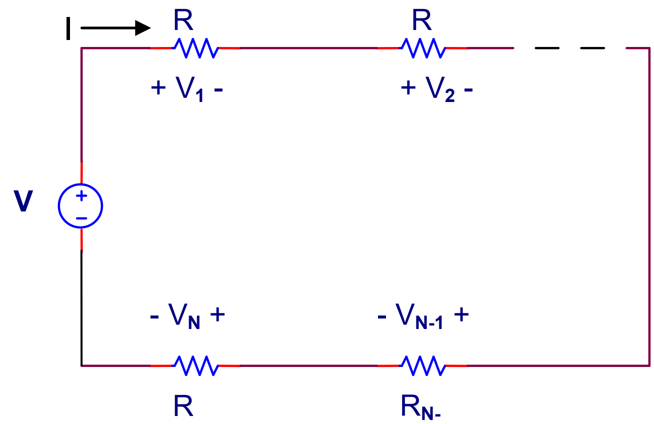

Figure 3. 1

The total series resistance R_S is given by

$$R_S = R_1 + R_2 + \cdots + R_{N-1} + R_N \qquad (3.3)$$

and

$$V_S = IR_S \qquad (3.4)$$

The Kirchhoff's voltage law indicates that:

$$V_S = V_1 + V_2 + \cdots + V_{N-1} + V_N \qquad (3.5)$$

The voltages across resistors can be obtained by multiplying the current by the corresponding resistors.

$$\left.\begin{array}{l} V_1 = IR_1 \\ V_2 = IR_2 \\ \vdots \\ V_{N-1} = IR_{N-1} \\ V_N = IR_N \end{array}\right\} \rightarrow \quad I = \frac{V_S}{R_S} \quad \rightarrow \begin{cases} V_1 = \left(\dfrac{R_1}{R_S}\right)V_S \\ V_2 = \left(\dfrac{R_2}{R_S}\right)V_S \\ \vdots \\ V_{N-1} = \left(\dfrac{R_{N-1}}{R_S}\right)V_S \\ V_N = \left(\dfrac{R_N}{R_S}\right)V_S \end{cases} \qquad (3.6)$$

The last expressions of equation 3.6 are known as voltage division.

Parallel Circuits:

In a parallel circuit the voltage is the same across all the elements.

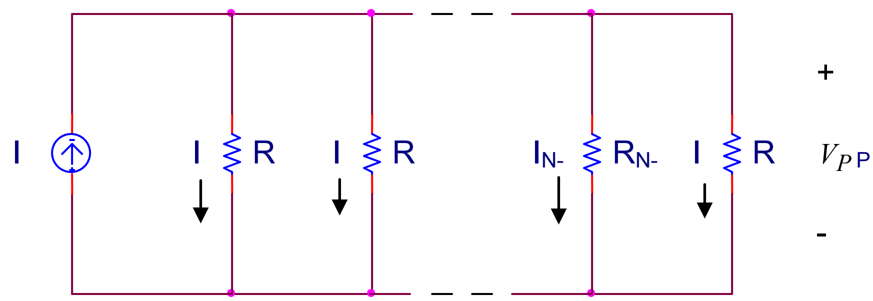

Figure 3. 2

The total parallel resistance, *Rp* is given by

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{N-1}} + \frac{1}{R_N} \qquad (3.7)$$

and

$$V_P = I_P R_P \qquad (3.8)$$

Kirchhoff's current law states:

$$I_P = I_1 + I_2 + \cdots + I_{N-1} + I_N \qquad (3.9)$$

The current through the branch resistors can be obtained by dividing the terminal voltage V_P by the corresponding branch resistance, R. therefore:

$$\left.\begin{array}{l} I_1 = \dfrac{V_P}{R_1} \\ I_2 = \dfrac{V_P}{R_2} \\ \vdots \\ I_{N-1} = \dfrac{V_P}{R_{N-1}} \\ I_N = \dfrac{V_P}{R_N} \end{array}\right\} \rightarrow \; V_P = I_P R_P \; \rightarrow \left\{\begin{array}{l} I_1 = \left(\dfrac{R_P}{R_1}\right) I_P \\ I_2 = \left(\dfrac{R_P}{R_2}\right) I_P \\ \vdots \\ I_{N-1} = \left(\dfrac{R_P}{R_{N-1}}\right) I_P \\ I_N = \left(\dfrac{R_P}{R_N}\right) I_P \end{array}\right. \qquad (3.10)$$

The last expressions of equation 3.10 are known as current division.

The reciprocal of resistance is known as conductance. It is expressed in the following equations:

$$G = \frac{1}{R} \qquad (3.11)$$

and

$$G_P = \frac{1}{R_P} \qquad (3.12)$$

This expression can be used to simplify equations 3.12 as shown below.

$$\left.\begin{aligned} I_1 &= G_1 V_P \\ I_2 &= G_2 V_P \\ &\vdots \\ I_{N-1} &= G_{N-1} V_P \\ I_N &= G_N V_P \end{aligned}\right\} \rightarrow V_P = \frac{I_P}{G_P} \rightarrow \left\{\begin{aligned} I_1 &= \left(\frac{G_1}{G_P}\right) I_P \\ I_2 &= \left(\frac{G_2}{G_P}\right) I_P \\ &\vdots \\ I_{N-1} &= \left(\frac{G_{N-1}}{G_P}\right) I_P \\ I_N &= \left(\frac{G_N}{G_P}\right) I_P \end{aligned}\right. \qquad (3.13)$$

where

$$G_P = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{N-1}} + \frac{1}{R_N} \qquad (3.14)$$

If only two resistors make up the network, as shown next

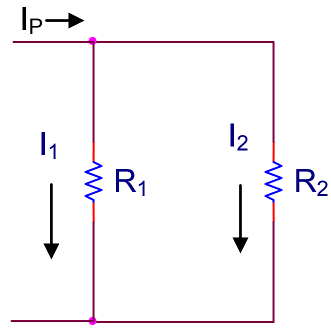

Figure 3.3

then the current in branches 1 and 2 can be calculated as follows:

$$I_1 = \left(\frac{G_1}{G_P}\right) I_P \qquad (3.15)$$

$$G_1 = \frac{1}{R_1} \qquad (3.16)$$

and

$$G_P = \frac{1}{R_1} + \frac{1}{R_2} \quad \rightarrow \quad G_P = \frac{R_1 + R_2}{R_1 R_2} \qquad (3.17)$$

$$\therefore \quad I_1 = \left(\frac{1}{R_1}\right)\left(\frac{R_1 R_2}{R_1 + R_2}\right) I_P \quad \rightarrow \quad I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_P \qquad (3.18)$$

In a similar fashion it can be shown that

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_P \qquad (3.19)$$

(Note how the current in one branch depends on the resistance in the opposite branch)

But, if the network consists of more than two resistors - say four

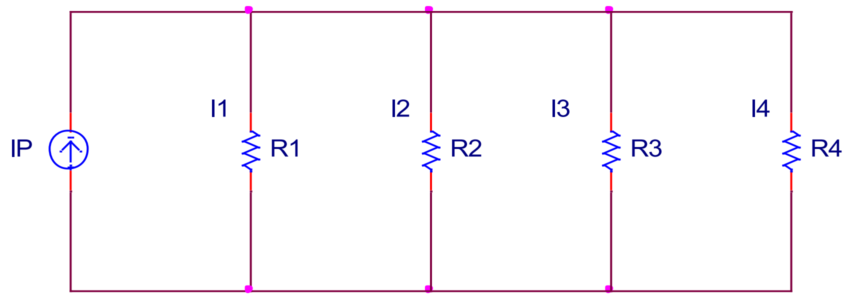

Figure 3.4

Then the calculation or branch currents using individual resistance becomes complex as demonstrated next, e.g.,

$$I_3 = \left(\frac{R_P}{R_3}\right) I_P \qquad (3.20)$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \quad \rightarrow \quad \frac{1}{R_P} = \frac{R_2R_3R_4 + R_1R_3R_4 + R_1R_2R_4 + R_1R_2R_3}{R_1R_2R_3R_4} \qquad (3.21)$$

so that

$$I_3 = \left(\frac{1}{R_3}\right)\left(\frac{R_1R_2R_3R_4}{R_2R_3R_4 + R_1R_3R_4 + R_1R_2R_4 + R_1R_2R_3}\right) I_P \qquad (3.22)$$

and

$$I_3 = \left(\frac{R_1R_2R_4}{R_2R_3R_4 + R_1R_3R_4 + R_1R_2R_4 + R_1R_2R_3}\right) I_P \qquad (3.23)$$

By using conductances, the above is simplified to

$$I_3 = \left(\frac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} \right) I_P \qquad (3.24)$$

and is easily accomplished with a hand calculator.

As the above demonstrates, when using current division, always use conductances and avoid using resistances in the calculation for all parallel networks with more than two resistors.

Series - Parallel Circuits

The analysis of series -parallel circuits is based on what has already been discussed. The solution of a series-parallel circuit with one single source usually requires the computation of total resistance, application of Ohm's law, Kirchhoff's voltage law, Kirchhoff's current law, voltage and current divider rules.

Preliminary Calculations:

Be sure to show all necessary calculations.

l. The resistors used in this lab all have 5% tolerances. This is denoted by the gold band. Calculate the minimum and maximum values for resistances with nominal values of 1kΩ and 2.7kΩ. Enter the values in Table 3.1.

2. Assume that the two resistors of problem 1 are used in the circuit of Figure 3.5. Calculate v_1, v_2, and I when R_1 and R_2 take on their minimum and maximum values and enter in Table 3.2.

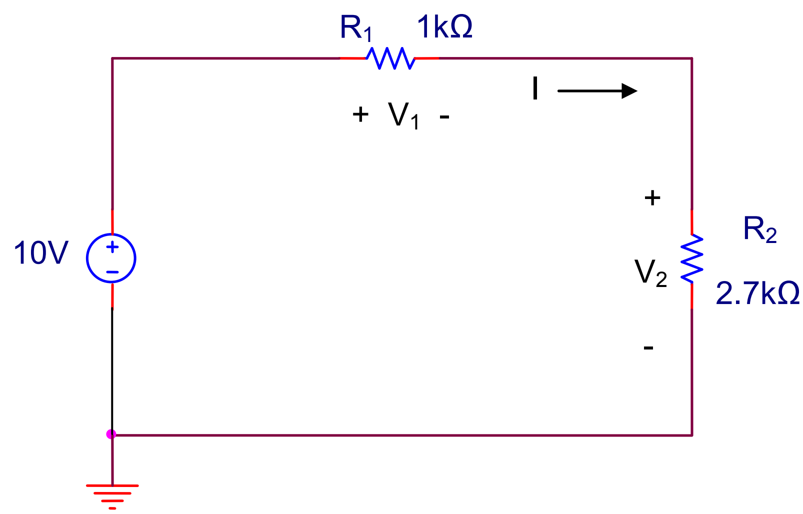

Figure 3.5

3. From your calculations in 2, record the maximum and the minimum possible values of I, v_1, and v_2 that you should see in the circuit in Table 3.3. Also, calculate and record the value of these variables when R_1 and R_2 are at the nominal values. What is the maximum % error in each of the variables possible due to the resistor tolerances?

4. For the circuit of Figure 3.6 calculate the resistance between nodes:

a. a and b (R_{a-b})

b. a and c (R_{a-c})

c. c and d (R_{c-d})

Enter your results in Table 3.4

Hint: Part c cannot immediately be reduced using series and parallel combinations.

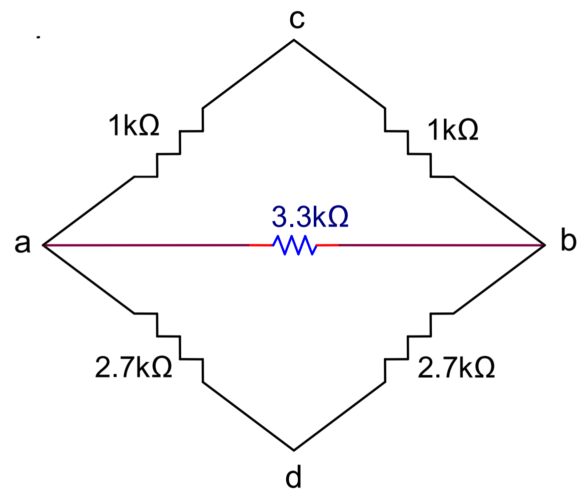

Figure 3.6

5. Use voltage division to calculate V_1 and V_2 for the circuit in Figure 3.7. Enter your results in Table 3.5.

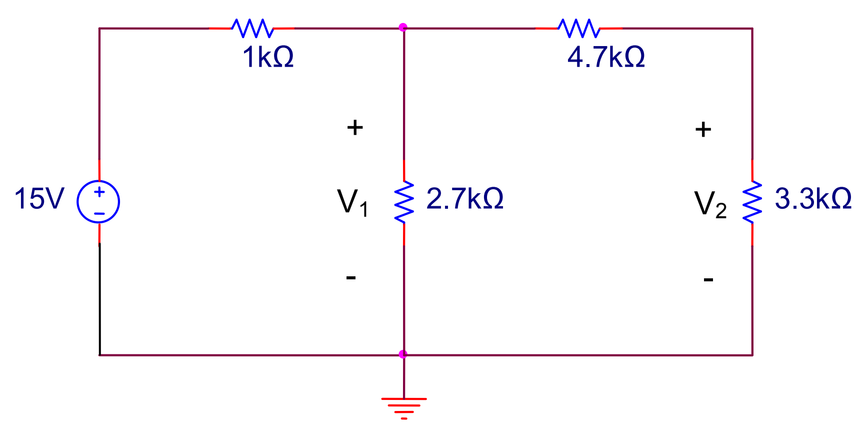

Figure 3.7

6. For the circuit in Figure 3.8, if R = 1k ohm, calculate I. Use current division to calculate I_R. Enter your results in Table 3.6. Repeat for R = 2.7k and 3.3k ohms.

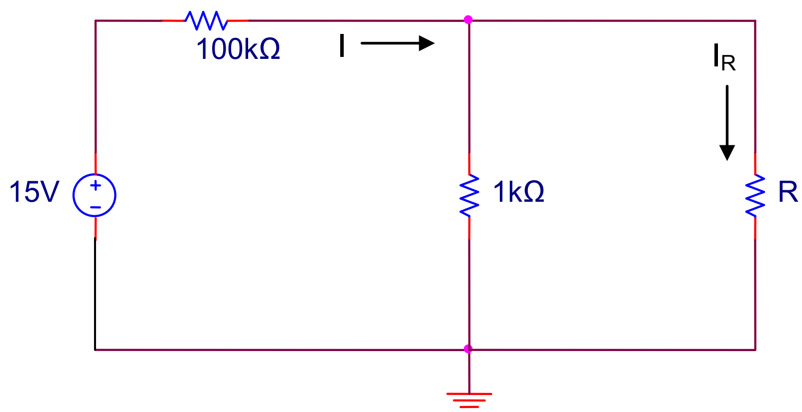

Figure 3.8

7. For the circuit in Figure 3.9, calculate each of the variables listed in Table 3.7.

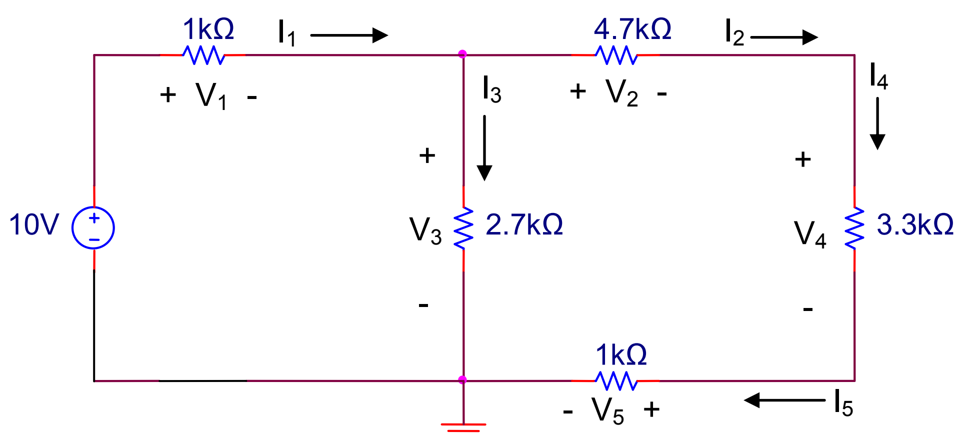

Figure 3.9

<u>Procedure</u>

l. Place a wire between the two measuring terminals of the ohmmeter and adjust the measurement reading to zero ohms. Obtain a 1kΩ and 2.7kΩ resistor and measure their values with the ohmmeter. What is the % error as compared to their nominal values? Enter your results in Table 3.1.

2. Construct the circuit in Figure 3.5. Measure V_1 and V_2 using the DMM only. Calculate I from your measurements. What is the % error as compared to their nominal values? Enter your results in Table 3.3.

3. Construct the circuit of Figure 3.6. Use an ohmmeter to measure the resistances listed in Table 3.4. Calculate the % error.

4. Construct the circuit of Figure 3.7. Measure V_1 and V_2 using the DMM only. Calculate the % error. Enter your results in Table 3.5.

5. Construct the circuit of Figure 3.8. Find I and I_R for R = 1kΩ, 2.7 kΩ, and 3.3 kΩ by measuring the appropriate voltages using the DMM only and applying Ohm's Law. Enter your results in Table 3.6. Note that I is approximately constant. Why?

6. Construct the circuit of Figure 3.9. Using the DMM, measure each of the variables listed in Table 3.7, and calculate the % error for each. Verify that KVL holds for each of the 3 loops in the circuit. Verify that KCL holds at each node. What can be said about $I_2 + I_3$ and I_1?

Table 3.1

Rnominal	Rmin	Rmax	Rmeas	% error
1kΩ				
2.7kΩ				

Table 3.2

	R_1,min R_2, min	R_1, max R_2, min	R_1, min R_2, max	R_1, max R_2,max
I				
V_1				
V_2				

Table 3.3

	max	min	nom	max % error	meas	% error
V_1						
V_2						
I						

Table 3.4

Resistance	Calculated	Measured	% error
R_{ab}			
R_{ac}			
R_{cd}			

Table 3.5

	Calculated	Measured	% error
V_1			
V_2			

Table 3.6

R	I, calc	I_R, calc	I, meas	I_R, meas
1kΩ				
2.7kΩ				
3.3kΩ				

Table 3.7

PARAMETER	CALCULATED	MEASURED	% ERR
V1			
V2			
V3			
V4			
V5			
I1			
I2			
I3			
I4			
I5			