ENM1500 Introductory Engineering Maths - Assignment 3

Weight: 10%. Due Date: Monday 14 October 2013.

Marks: 100

Notes

- The due date is the date by which you must submit your assignment to USQ. You must submit a scanned copy of your assignment through EASE by 11:55 pm on the due date to avoid penalty. The due date will be determined according to USQ time, not the timezone from which you may be submitting your assignment. **Submit a readable copy of your assignment as a single document in PDF format**.
- Writing by hand is quickest and best for this course. If you choose to typeset your solutions, take care to get the notation and symbols correct. Note that you *do not* need to set it out in report format (with aim, method, results, etc.), there is no particular style that we demand.
- Show all working and steps in algebra. Always offer clear and logical working that is easy to follow. Give the marker every opportunity to see how you obtained your answer, as we are assessing your reasoning as well as the final result. Where appropriate, check your answer to make sure it is correct.
- Graphs should always have a title and axis labels with units if appropriate. Neatly hand drawn figures *of a reasonable size* are fully acceptable. You may use a graphics calculator or computer, but make sure to interpret the output clearly and write your conclusion. Offer careful scans of any hand-copied plots you submit with your assignment. Do not neglect the scale and labels and check the readability of your scanned work.

Question 1 [15 marks]

A radar station located at (0,0) makes two radar contacts. Contact **a** is made 14° east of north at 7.6 km. Another radar contact, **b** is made 27° west of south at 5.1 km.

4 marks (a) Sketch a scale diagram of both radar contacts, **a** and **b**.

6 marks (b) How far is **b** from **a**?

5 marks (c) At what heading relative to north is contact **a** from contact **b**?

Question 2 [10 marks]

A robotic arm moves a component in 3D space from position **a** at (1,2,1) m to position **b** at (3,3,2) m. The force required to move the component can be described by the force vector (60,-10,30) N.

4 marks (a) Find the displacement vector that describes the movement of the component from position **a** to position **b**. Give your answer in $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ notation.

6 marks (b) What work is done by the robotic arm in moving the component?

Question 3 [20 marks]

Consider a firework that explodes into four fragments. The velocity of each fragment in m/s can be described by the following vectors:

$$\mathbf{a} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 1\hat{\mathbf{k}}$$
$$\mathbf{b} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$
$$\mathbf{c} = 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$
$$\mathbf{d} = 2\mathbf{a}$$

7 marks (a) What is the magnitude of fragment **d**?

3 marks (b) Find the angle between fragments **a** and **b**.

4 marks (c) Find the angle between fragments **a** and **c**.

6 marks (d) What is the projection of fragment **c** in the direction of **a**?

Question 4 [14 marks]

Given the matrices:

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & -1 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 4 & 2 \\ 1 & 2 & -2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 0 & 1 \\ 4 & 5 & 1 \end{bmatrix}$$

Find:

3 marks (a) $4\mathbf{B}$

4 marks (b) $\mathbf{A}^{\mathsf{T}}\mathbf{A}$

4 marks (c) $\mathbf{BC}$

3 marks (d) $\mathbf{A} - \mathbf{C}$

Question 5 [16 marks]

Three orders from a sand and gravel business are filled in a single day. The orders are as follows:

Order 1
$14\,\mathrm{m}^3$ of sand, $2\,\mathrm{m}^3$ of cement and $7\,\mathrm{m}^3$ of gravel for \$1700

Order 2
$4\,\mathrm{m}^3$ of sand, $4\,\mathrm{m}^3$ of cement and $9\,\mathrm{m}^3$ of gravel for \$1200

Order 3
$7\,\mathrm{m}^3$ of sand, $10\,\mathrm{m}^3$ of cement and $1\,\mathrm{m}^3$ of gravel for \$1500

4 marks (a) Express each order and its associated total cost as a matrix equation of the form:

$$\mathbf{AX} = \mathbf{B} \tag{1}$$

where $\mathbf{A}$ is a matrix of each order's components, X is a matrix of costs for sand, cement and gravel and $\mathbf{B}$ is a matrix representing the total cost for each order.

6 marks (b) Calculate the inverse of matrix $\mathbf{A}$. *(Hint: Use MS Excel)*

6 marks (c) Using the calculated inverse matrix of $\mathbf{A}$, solve matrix equation 1 and hence determine the cost of sand, cement and gravel per m^3.

Question 6 [10 marks]

Consider the polynomial function:

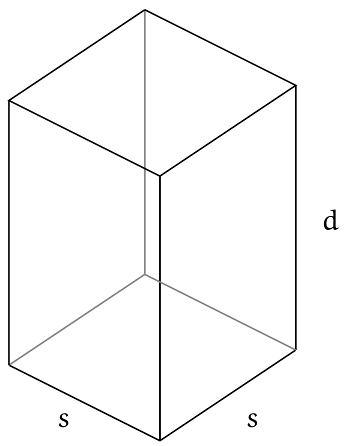
$$f(x) = 2x^4 - 10x^3 + x$$

4 marks (a) What is the derivative of the function, $f'(x)$?

6 marks (b) Evaluate derivative, $f'(x)$ at $x = 0$, $x = 3$ and $x = 4$

Question 7 [15 marks]

An underground storage facility is being built to contain waste. The facility consists of a shaft that is to be lined with spray on concrete. The shaft is constructed to utilise the maximum allowable surface area of spray on concrete of $310\,\mathrm{m}^2$ and must have a square base as shown below:



3 marks (a) Provide an equation for the total internal surface area that needs to be lined with spray on concrete given that the shaft is open and has no top.

3 marks (b) Rearrange your equation for internal surface area to make the shaft depth, d the subject of the equation.

9 marks (c) Shaft volume, V can be expressed as:

$$V = s^2 d \tag{2}$$

Using equation 2, and your expression for d in terms of shaft surface area and depth from part (b), calculate the maximum volume able to be contained within the newly constructed shaft if the maximum internal spray on surface area is $310\,\mathrm{m}^2$.

Total: 100 marks

End of Assignment.