Due: Thu Feb 28 2013 **MTH 437 Homework 1**

Problem 1 (10 points)

Consider the n-th Taylor polynomial T_n of a function f given by

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

1. Compute the 4-th Taylor polynomial for e^x, $\sin(x)$, $\cos(x)$.
2. Use matlab to find the maximum error $\|f(x) - T_4(x)\|_\infty$ for the three functions above and their 4-th order Taylor polynomial in $[-\pi/6, \pi/6]$. Print out the code that you used.
3. Verify at this order the formula

$$e^{ix} = \cos(x) + i\sin(x)$$

Problem 2 (10 points)

Consider the following differential equation

$$\dot{N} = -\alpha N, \qquad N(0) = N_0$$

1. Find the analytical solution of the equation.
2. Find the time t_h such that $N(t_h) = N_0/2$.
3. Write a matlab code using the Euler-scheme that solves the above equation for $N_0 = 10$, $\alpha = 0.1$ $dt = 0.1$. Print out the code itself and a graph of the solution of your code in $[0, 10]$. What is the maximum error of the numerical solution and the analytical solution in this interval?
4. Repeat the above steps using the mid-point method.

Problem 3 (10 points)

Repeat the steps in problem 2 for the logistic model

$$\dot{N} = rN\left(1 - \frac{N}{K}\right)$$

You can use the analytical solution discussed in class. Find appropriate parameters r and K.