EE 5322 Homework 3
Discrete Time Simulation, Observers, Kalman Filter

1. Discrete-Time System.

A discrete time system is given by

$$x_{k+1} = Ax_k + Bu_k, \quad x_0$$

Write a MATLAB m file to simulate the system, i.e. to compute x_k for a given input u_k, initial condition x_0, and range of the time index *k= 1,2,...,N.*

a. Simulate the system

$$x_{k+1} = \begin{bmatrix} 0 & 1 \\ -0.89 & 1.8 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k$$

for u_k equal to the unit step and x_0=0. Plot x_k vs. k for 100 time samples.

Find the period and percent overshoot.

b. Simulate the same system but now add process noise so that

$$x_{k+1} = Ax_k + Bu_k + w_k.$$

Take the noise w_k as uniformly distributed between 0 and 0.2. Use MATLAB function rand. Plot x_k vs. k for 100 time samples.

2. DT Kalman Filter

Write a MATLAB m file to simulate a DT system

$$x_{k+1} = Ax_k + Bu_k + Gw_k$$

$$z_k = Hx_k + v_k$$

plus DT Kalman Filter.

Simulate the optimal time-varying DT Kalman Filter for the system

$$x_{k+1} = \begin{bmatrix} 0 & 1 \\ -0.89 & 1.8 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k + w_k,$$

$$z_k = [1 \quad 0] x_k + v_k.$$

Take the process noise w_k as a normal 2-vector (MATLAB function randn) with each component having zero mean and variance 0.1. Take measurement noise v_k as normal (0,0.1). Use u_k equal to the unit step and x_0=0.

Plot the states and their estimates on the same graphs. i.e. $x_1(k), \hat{x}_1(k)$ on one graph, and $x_2(k), \hat{x}_2(k)$ on another graph.

3. Steady-state DT Kalman Filter

a. Find the steady-state Kalman gain by iteration on the time-varying Riccati difference equation.

b. Find the steady-state Kalman gain by solution of the ARE using dlqe in MATLAB.

c. Simulate the system in problem 2 with the steady-state Kalman Filter, which has a constant gain. Compare the results with the optimal Kalman filter in problem 2.