1. Define a partition of $X = \mathbb{R}^2 - \{O\}$ by taking each ray emanating from the origin as an element in the partition. (See Figure 3.25.) Which topological space that we have previously encountered appears to be topologically equivalent to the quotient space that results from this partition?

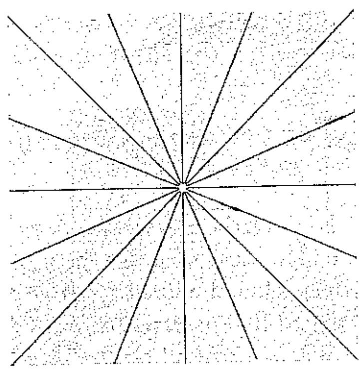

FIGURE 3.25: A partition of $X = \mathbb{R}^2 - \{O\}$.

2 Provide an example showing that a quotient space of a Hausdorff space need not be a Hausdorff space.

3 Consider the equivalence relation on $\mathbb{R}^2$ defined by $(x_1, x_2) \sim (w_1, w_2)$ if $x_1^2 + x_2^2 = w_1^2 + w_2^2$. Describe the quotient space that results from the partition of $\mathbb{R}^2$ into the equivalence classes in this equivalence relation.

4 In each of the following cases, describe or draw a picture of the resulting quotient space. Assume that points are identified only with themselves unless they are explicitly said to be identified with other points.

- **(a)** The disk with its boundary points identified with each other to form a single point.
- **(b)** The circle S^1 with each pair of antipodal points identified with each other.
- **(c)** The interval $[0, 4]$, as a subspace of $\mathbb{R}$, with integer points identified with each other.
- **(d)** The interval $[0, 9]$, as a subspace of $\mathbb{R}$, with even integer points identified with each other to form a point and with odd integer points identified with each other to form a different point.
- **(e)** The real line $\mathbb{R}$ with $[-1, 1]$ collapsed to a point.
- **(f)** The real line $\mathbb{R}$ with $[-2, -1] \cup [1, 2]$ collapsed to a point.
- **(g)** The real line $\mathbb{R}$ with $(-1, 1)$ collapsed to a point.
- **(h)** The plane $\mathbb{R}^2$ with the circle S^1 collapsed to a point.
- **(i)** The plane $\mathbb{R}^2$ with the circle S^1 and the origin collapsed to a point.
- **(j)** The sphere with the north and south pole identified with each other.
- **(k)** The sphere with the equator collapsed to a point.

5 **(a)** Show that a hexagon with opposite edges glued together straight across yields a torus.
(b) Show that a hexagon with opposite edges glued together with a flip yields a projective plane.

6. Give a representation of $T\#P$, the connected sum of a torus and a projective plane, as a hexagon with pairs of edges glued together.

7 Show that the quotient space in Example 3.27 is topologically equivalent to $S^1 \times P$, the product of a circle and a projective plane.

EXAMPLE 3.27. Consider the quotient space obtained by identifying opposite faces of a cube, as shown in Figure 3.37. If we take a line segment running from the center of the left face of the cube to the center of the right face, then when we glue the space together, the segment becomes a circle. If we take a cylinder centered on that segment, then when we glue the space together, the circles at the ends of the cylinder are glued together with a flip, resulting in a Klein bottle. Therefore, within the space there is a collection of concentric Klein bottles that shrinks down to a core circle.

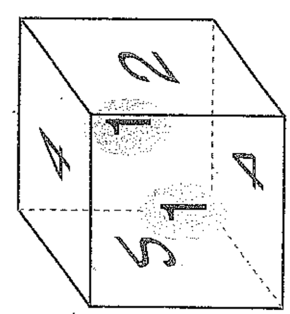

FIGURE 3.37: Gluing the faces of a cube to obtain $S^1 \times P$.

Information for Q. 7.