1 Intertemporal trade and durable consumption goods

Suppose that a representative individual in a small open economy lives for infinite periods. The individual enters each period $t(t = 0, 1, 2, ...)$ holding foreign assets B_t, capital stock K_t and durable goods stock D_t. The initial foreign assets B_0 and capital stock K_0 are given, and the initial durable goods stock is zero, $\bar{D}_0 = 0$. The individual produces one tradable and non-durable good during each period according to the production function

$$Y_t = F(K_t)$$

where Y_t is the output in terms of non-durable goods. The production function satisfies the standard assumptions, and there is no capital depreciation.

One tradable and durable good can be produced from non-durable goods at a constant price p in terms of nondurable goods. The stock of durable goods at the end of each period t, denoted by D_t, includes the durable goods stock left from the previous period and the new purchase of the durable good in period t. The durable goods stock provides a proportional consumption service flow each period and depreciates at a constant rate δ each period. At the end of each period t, the individual allocates her resources between non-durable goods C_t, durable goods D_t, investment I_t and foreign assets B_{t+1}. The prices of foreign assets, non-durable goods and capital goods are all normalized to one.

The individual is to maximize her lifetime utility

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, D_t)$$

where $\beta(0 < \beta < 1)$ is the subjective discount factor and the period utility function is

$$u(C_t, D_t) = \gamma \ln C_t + (1 - \gamma) \ln D_t, 0 < \gamma < 1$$

The world interest rate is constant at r and satisfies $\beta = 1/(1 + r)$.

(1) Show the timing of events in a diagram.

(2) How much is the expenditure on durable goods in period t?

(3) Write down the period budget constraint.

(4) What is the constraint on the terminal foreign assets?

(5) Write down the individual's optimization problem in terms of the period budget constraint.

(6) Derive the first-order conditions.

(7) Write down the intertemporal budget constraint.

(8) Write down the individual's optimization problem in terms of the intertemporal budget constraint.

(9) Derive the first-order conditions.

(10) Interpret the first-order conditions.

(11) How are investment and output determined?

(12) Solve the optimal consumption for durable goods and non-durable goods.

(13) How much is the purchase of durable goods each period in a special case where the durable goods do not deteriorate ($\delta = 0$)?

(14) How much is the purchase of durable goods each period in a special case where the durable goods completely deteriorate ($\delta = 1$)?

(15) How much is the purchase of durable goods each period in a special case where the durable goods partially deteriorate ($\delta = 0.1$)?

(16) Is the model result in the above case of $\delta = 0.1$ consistent with individual behaviors in the real world? Explain why.

(17) Derive the current account in the initial period in terms of permanent levels of variables.

(18) Interpret this current account equation.

(19) Show the current account in the case where, ceteris paribus, durable goods become non-durable, is a special case of the above current account equation.

(20) What is the effect of durable goods on the current account based on the above equation?