Homework #3

1. **Problem 1**

[a] $\mathbf{V} = 100\angle -45^\circ \text{ V}, \qquad \mathbf{I} = 20\angle 15^\circ \text{ A}$

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45-(15)] = 500 \text{ W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000 \sin -60^\circ = -866.03 \text{ VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\angle -45^\circ, \qquad \mathbf{I} = 20\angle 165^\circ$

$$P = 1000\cos(-210^\circ) = -866.03 \text{ W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500 \text{ VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100\angle -45^\circ, \qquad \mathbf{I} = 20\angle -105^\circ$

$$P = 1000\cos(60^\circ) = 500 \text{ W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03 \text{ VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100\angle 0^\circ, \qquad \mathbf{I} = 20\angle 120^\circ$

$$P = 1000\cos(-120^\circ) = -500 \text{ W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-120^\circ) = -866.03 \text{ VAR}, \qquad \text{B} \rightarrow \text{A}$$

2. **Problem 2**

For $v(t) = 3\cos(\omega t + 25^o)$ and $i(t) = 5\sin(\omega t - 65^o)$, compute the power factor and reactive factor.

Ans) PF= $\cos(\theta_v - \theta_i) = \cos(25^o - (-155^o)) = \cos(180^o)) = -1$

RF = $\sin(\theta_v - \theta_i) = \sin(25^o - (-155^o)) = \sin(180^o)) = 0$

3. **Problem 3**

From Ex. 9.4 $I_{\text{eff}} = \dfrac{I_\rho}{\sqrt{3}} = \dfrac{0.18}{\sqrt{3}} \text{ A}$

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54 \text{ W}$$

4. **Problem 4 (The problem means that a capacitor of -j52 is connected to the load in parallel)**

[a] $Z = (39 + j26) \| (-j52) = 48 - j20 = 52\angle -22.62^\circ \ \Omega$

Therefore $\mathbf{I}_\ell = \dfrac{250\angle 0^\circ}{48 - j20 + 1 + j4} = 4.85\angle 18.08^\circ \text{ A(rms)}$

$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\angle -22.62^\circ)(4.85\angle 18.08^\circ) = 252.20\angle -4.54^\circ \text{ V(rms)}$

$\mathbf{I}_\text{L} = \dfrac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\angle -38.23^\circ \text{ A(rms)}$

[b] $S_L = \mathbf{V}_L \mathbf{I}_L^* = (252.20\angle -4.54^\circ)(5.38\angle +38.23^\circ) = 1357\angle 33.69^\circ$

$= (1129.09 + j752.73)\,\text{VA}$

$P_L = 1129.09\,\text{W}; \qquad Q_L = 752.73\,\text{VAR}$

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52\,\text{W}; \qquad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09\,\text{VAR}$

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)\,\text{VA}$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_{\text{cap}} = \dfrac{|\mathbf{V}_L|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18\,\text{VAR}$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

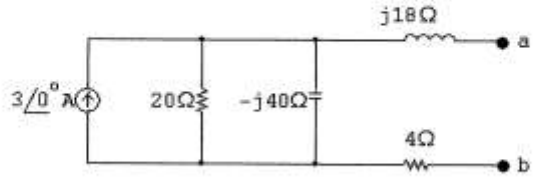
Check: $94.09 + 752.73 + 376.36 = 1223.18\,\text{VAR}$ and

$1129.09 + 23.52 = 1152.62\,\text{W}$

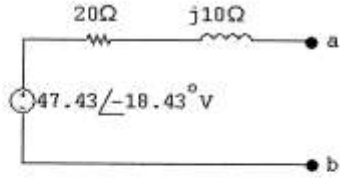
5. Problem 5

[a] The phasor domain equivalent circuit and the Thévenin equivalent are shown below:

Phasor domain equivalent circuit:



Thévenin equivalent:



$\mathbf{V}_{\text{Th}} = 3\dfrac{-j800}{20 - j40} = 48 - j24 = 53.67\angle -26.57^\circ\,\text{V}$

$Z_{\text{Th}} = 4 + j18 + \dfrac{-j800}{20 - j40} = 20 + j10 = 22.36\angle 26.57^\circ\,\Omega$

For maximum power transfer, $Z_L = (20 - j10)\,\Omega$

[b] $\mathbf{I} = \dfrac{53.67\angle -26.57^\circ}{40} = 1.34\angle -26.57^\circ\,\text{A}$

Therefore $P = \left(\dfrac{1.34}{\sqrt{2}}\right)^2 20 = 17.96\,\text{W}$

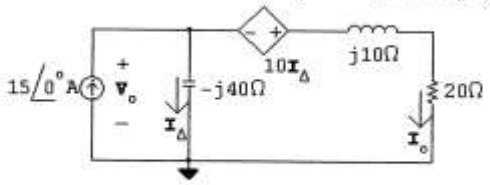
[c] $R_L = |Z_{\text{Th}}| = 22.36\,\Omega$

[d] $\mathbf{I} = \dfrac{53.67\angle -26.57^\circ}{42.36 + j10} = 1.23\angle -39.85^\circ\,\text{A}$

Therefore $P = \left(\dfrac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17\,\text{W}$

6. Problem 6

$$j\omega L = j10{,}000(10^{-3}) = j10\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j10{,}000(2.5)} = -j40\,\Omega$$



$$-15 + \frac{\mathbf{V}_o}{-j40} + \frac{\mathbf{V}_o + 10(\mathbf{V}_o/-j40)}{20 + j10} = 0$$

$$\therefore \quad \mathbf{V}_o\left[\frac{1}{-j40} + \frac{1 + j0.25}{20 + j10}\right] = 15$$

$$\therefore \quad \mathbf{V}_o = 300 - j100\,\text{V}$$

$$\therefore \quad \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j40} = 2.5 + j7.5\,\text{A}$$

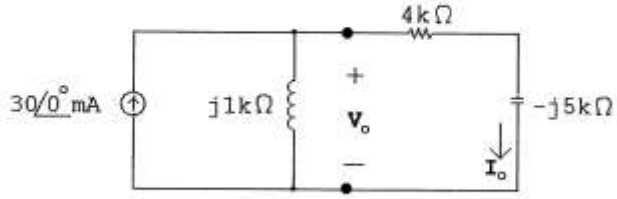
$$\mathbf{I}_o = 15\underline{/0^\circ} - \mathbf{I}_\Delta = 15 - 2.5 - j7.5 = 12.5 - j7.5 = 14.58\underline{/-30.9^\circ}\,\text{A}$$

$$P_{20\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 20 = 2125\,\text{W}$$

7. Problem 7

$$\mathbf{I}_g = 30\underline{/0^\circ}\,\text{mA}$$

$$j\omega L = j(100)(10) = j1000\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j(100)(2)} = -j5000\,\Omega$$



$$\mathbf{I}_o = \frac{30\underline{/0^\circ}(j1000)}{4000 - j4000} = 3.75\sqrt{2}\underline{/135^\circ}\,\text{mA}$$

$$P = |\mathbf{I}_o|^2_{\text{rms}}(4000) = (3.75)^2(4000) = 56.25\,\text{mW}$$

$$Q = |\mathbf{I}_o|^2_{\text{rms}}(-5000) = -70.3125\,\text{mVAR}$$

$$S = P + jQ = 56.25 - j70.3125\,\text{mVA}$$

$$|S| = 90.044\,\text{mVA}$$