

MAT 120

Name: _____

Test 2

(v1)

Please be neat. Sloppiness, disorganization, etc. may, in the aggregate, result in a point reduction. Please show all work. No credit for correct final answer without a valid argument. Show your work graphically in all relevant questions. Use the formula, substitution, answer method whenever possible.

1. The *amazing car dealer* in Greater Portland area has compiled the sales data for the past five years and developed the following probability distribution:

| | | | | | | |
|--------|------|-----|------|-----|-----|-----|
| x | 10 | 11 | 12 | 13 | 14 | 15 |
| $p(x)$ | 0.30 | .25 | 0.10 | .15 | .15 | .05 |

where X is the number of cars sold per day.

(i) (2 pts.) What is the probability that at least 12 cars are sold in a given day?

(ii) (2 pts.) What is the expected value of X (i.e. the mean sales per day).

(iii) (2 pts.) What is the standard deviation of X .

2. (2 pts.) Statistics indicate that alcohol is a factor in about 60% of fatal automobile accidents. Of the next 6 accidents, find the probability that alcohol is a factor in at least 5, that is $p(X \geq 5)$. Use the binomial formula.

3. (2 pts.) Suppose that each item produced is independently defective with probability 0.001. Find the *mean* and *standard deviation* of the number of defective items produced in a shipment of size 5000. (No formula, no credit).

4. (2 pts.) Suppose that patients arrive at an emergency room at a mean rate of 5 patients per hour. Using the Poisson model find the probability that 3 patients arrive during one hour.

5. (2 pts.) Let Z be a standard normal distribution. Find z_0 such that $p(Z \geq z_0) = 0.07$, that is $z_{.07}$. (Show your work graphically).

6. The height of adult women in the United States, X , is normally distributed with mean 64.5 inches and standard deviation 2.4 inches.

(i) (2 pts.) Alice is 72 inches tall. What percentage of women are taller than Alice.

7. The time, X , in hours required by a mechanic to repair a machine has an exponential distribution with $\lambda = 0.5$.

(i) (2 pts.) What is the probability that the time until the machine is repaired exceeds 2.5 hours?

8. The number of trips to doctor's office per family per year in a given community, X , is known to have a mean of 12 with a standard deviation of 3. Suppose a random sample of 36 families is taken and a sample mean, \bar{X} , is calculated.

(i) (2 pts.) Describe the sampling distribution of the sample mean, \bar{X} . (Include the mean $\mu_{\bar{X}}$, standard deviation $\sigma_{\bar{X}}$, and type of distribution).

(ii) (2 pts.) Find the probability that the sample mean, \bar{X} , does not exceed 13, i.e. $P(\bar{X} \leq 13)$.

9. A local pizza parlor advertises that their average time for delivery of a pizza is within 30 minutes of receipt of the order. To check the accuracy of their claim, the delivery time for a random sample of 64 orders were recorded, with a sample mean of 34 minutes and a standard deviation of 21 minutes.

(i) (2 pts.) Find a 99% confidence interval for the mean delivery time i.e. the population mean.

(iii) (2 pts.) Find the sample size necessary to reduce the width of the confidence interval in (i) by half.

10. (4 pts.) Answer by True or False . (Circle your choice).

T F (i) The only rule that applies to all probability distributions is that the possible random variable values are always between 0 and 1.

T F (ii) The central limit theorem applies regardless of the shape of the original population frequency distribution.

T F (iii) The standard normal distribution has its mean equal to one and standard deviation equal to zero.

T F (iv) As the sample size increases, the width of the confidence interval for the population mean tends to decrease.