The University of Texas at Austin
Department of Electrical and Computer Engineering

EE362K: Introduction to Automatic Control—Spring 2016

Problem Set Four

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This problem tests more linear algebra, and introduces reachability which we just began last week.

1. Read Chapter 6.3, 7.1 of the text.
2. Eigenvalue Assignment: Exercise 6.9 from the book.
3. Second Order Systems and State Feedback Design: Thus far, we have largely focused on stability as our performance metric of choice. Yet once stability is assured, we can focus on finer details. A few classes ago, we discussed other system properties, such as rise time, overshoot, and settling time. In this exercise, we will explore how controlling eigenvalues through Eigenvalue assignment is important beyond just placing them in the OLHP to insure stability.

   Read the beginning of Section 6.3 on second order systems. Replicate the computations in Example 6.6 exploring the tradeoff of overshoot and rise time by plotting the curves for smaller values of ζ (the book shows the curve for $\zeta = 0.9$).
4. We introduced the notion of an observable system (see also Chapter 7.1). Which of the following is observable:

   (a)
$$A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & -2 & 1 \\ 0 & 0 & 6 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$
$$A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & -2 & 1 \\ 0 & 0 & 6 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$
$$A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & -2 & 1 \\ 0 & 0 & 6 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

   (b)
$$A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$
$$A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

$$A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix}, \qquad C = [\ 1 \quad 0 \quad 0\ ].$$

5. For the cases of the previous problem above where the pair (A, C) is observable, find the observer canonical form of each pair, (A, C).

6. Consider the system

$$\dot{x} = Ax, \qquad y = Cx,$$

where

$$A = \begin{bmatrix} 3 & 1 & 0 \\ -3 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \qquad C = [\ 1 \quad 0 \quad 0\ ].$$

Suppose that $x(0) = (1\ 1\ 1)$. Note that the pair (A, C) is already in OCF, and hence the problem is observable (but go ahead and check for yourselves). In this problem, you will design a *linear observer* as we described in class.

Define a (dummy) variable $\hat{x}$, with $\hat{x}(0) = (0\ 0\ 0)$ and dynamics

$$\dot{\hat{x}} = A\hat{x} + L(y - \hat{y}), \qquad \hat{y} = C\hat{x}.$$

We would like for $\hat{x}$ to be equal to x, but the trouble is: what if we can only observe $y = Cx$, and we do not know that $x(0) = (1\ 1\ 1)$? One answer is to build a *linear observer* – a dummy system that tries to converge to the true value of x, but without knowledge of $x(0)$, and using only y.

   (a) Consider the variable $\tilde{x} = x - \hat{x}$. This is the error in our estimation of x. Write the dynamics for $\tilde{x}$. That is, write down $\dot{\tilde{x}}$ (you should have only $\tilde{x}$ on the right hand side, i.e., no x, y, $\hat{x}$ or $\hat{y}$.

   (b) Design a 3×1 matrix L so that $\tilde{x}$ converges to zero. This means that $\hat{x}$ converges to x.

   (c) Now plot $\tilde{x}$ over time to show that indeed it goes to zero. Also, plot x and $\hat{x}$ over time, to see that $\hat{x}$ converges to x.

7. Show that the characteristic polynomial of the matrix

$$A = \begin{bmatrix} -a_{n-1} & 1 & \cdots & 0 \\ -a_{n-2} & & \ddots & 0 \\ \vdots & & & 1 \\ -a_0 & & & 0 \end{bmatrix}$$

is

$$p_A(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_0.$$

One way to proceed is by induction on the size of the matrix, as we did in class.

8. (Optional) Cayley-Hamilton Theorem: Exercise 6.10 from the book. Recall that we had used this result in order to show that the reachability matrix does not increase its rank after we have added the n^{th} column $A^{n-1}B$, where n is the dimension of the system.