## FINAL EXAM Summer 2014

Name:

20 questions, 180 minutes absolute max. **No book or notes.** A normal scientific calculator is ok, but **no graphing calculators/no mobile devices.** 

$a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2})$	f(x) + b = shift up b
$a^{3} + b^{3} = (a + b)(a^{2} - ab + b^{2})$	f(x+b) = shift left b
$y = a(x - h)^2 + k$	bf(x) = vertical stretch by factor of b
$y = Ae^{kt}$	f(bx) = horizontal compression by factor of 1/b
$\mathbf{y} = \mathbf{A}(1 - \mathbf{e}^{-\mathbf{kt}})$	-f(x) = reflection over x-axis f(-x) = reflection over y-axis

1. Simplify using appropriate laws or exponents. Any fractions should be reduced and exponents should be positive:



2. Combine into a single logarithm:

$$ln(5) + \frac{1}{2}ln(x) - 3ln(y + 1)$$

3. Rewrite in simplified radical form:  $(16x^5)^{-2/3}$ 

4. Find all solutions, both real and complex:  $x^5 - x^3 - x^2 + 1 = 0$ 

5. Find all real solutions, if any:

$$\sqrt{x-4} + \sqrt{x+4} = 2\sqrt{x-1}$$

6. Solve using a switch of variables:

$$\left(\frac{2y+1}{y+3}\right)^2 + \frac{8y+4}{y+3} = -4$$

7. Freddie's Fruits delivers orchard-fresh fruits to your doorstep every morning. If you sign up for a 5.00/month VIP membership, Freddie will deliver up to fifty fruits at the amazingly reasonable price of only 85¢ each (additional fruits may be purchased at 90¢ each). Write a function for the cost C of purchasing x fruits from Freddie in a given month.

8. Given g(x) = sqrt(x - 1) + 4,

- a) identify the parent function.
- b) list the transformations being applied to the parent function, in a correct order.
- c) Sketch a graph of g, labeling the coordinates of at least three points.

9. Given g(x) = 4 - |1 - x|

- a) identify the parent function.
- b) list the transformations being applied to the parent function, in a correct order.
- c) Sketch a graph of g, labeling the coordinates of the vertex and at least two other points.

10. A parabola has its vertex at (1, 2) and passes through the point (-2/3, 3/4). Find its equation in general form.

11. Biff is getting ready to harvest his bumbleberries. If he harvests now, he will reap about 80 bushels per acre and they'll sell for \$24 each. Each week that he waits, the birds will eat about 3 bushels per acre, but the price will increase by \$1.30 per bushel. When should Biff harvest his bumbleberries if he wants to maximize revenue?

12. z varies directly with the square of x and inversely with the cube of y. If z = 12 when x = 2 and y = -3, what value does z take when x = -3 and y = 2?

13. Given  $f(x) = \frac{x+4}{3x}$  and  $g(x) = \frac{x+4}{x-2}$  construct (g/f)(x) and state its domain.

14. Given  $g(x) = \frac{x+4}{x-2}$  construct  $g \circ g$  and state its domain.

15. Verify algebraically that g is the inverse of f:

$$f(x) = \frac{2x-3}{3x+1}$$
  $g(x) = \frac{x+3}{2-3x}$ 

16. In the year 1974, 80 of Selma's puppies got sick. If the number of sick puppies has been increasing steadily by 3% per year, how many sick puppies will Selma have in the year 2020?

17. Biffium-X has a half-life of 340 years. If you start with a 40-gram chunk of this stuff, how much will remain after 150 years?

18. Sketch a graph of  $y = \log_3(x)$ , clearly labeling the coordinates of the Five Amigos. Then, apply transformations to sketch a graph of  $y = \log_3(2x - 5)$ . Be sure to show where the Five Amigos (and asymptote!) end up after undergoing the transformations.

19. Solve for x, expressing your answer in exact form: log(x) - ln(x) = 1

20. Zeke is memorizing 100 very interesting math facts. If the first 40 took him an hour to learn, how much longer will it take him to learn another 40? Round your answer to the nearest tenth of a minute.