MATH 107 FINAL EXAMINATION

This is an open-book exam. You may refer to your text and other course materials as you work on the exam, and you may use a calculator. **You must complete the exam individually. Neither collaboration nor consultation with others is allowed.**

Record your answers and work on the separate answer sheet provided.

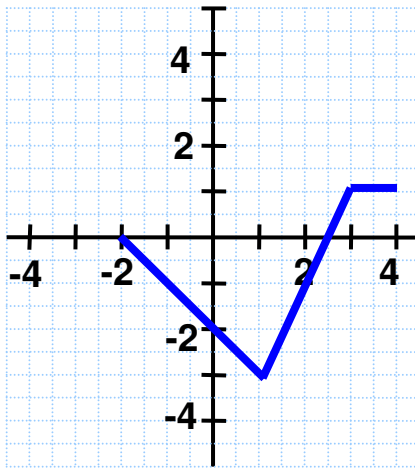There are 30 problems.
Problems #1–12 are Multiple Choice.
Problems #13–21 are Short Answer. (Work not required to be shown)
Problems #22–30 are Short Answer with work required to be shown.

MULTIPLE CHOICE

1. Determine the domain and range of the piecewise function.   1. _____

    A.   Domain [– 3, 1]; Range [– 2, 4]
    B.   Domain [– 2, 2.5]; Range [–3, 0]
    C.   Domain [– 2, 4]; Range [– 3, 1]
    D.   Domain [–2, ∞); Range [–3,∞)

2. Solve: $\sqrt{3x-5} = x-3$ 2. _____
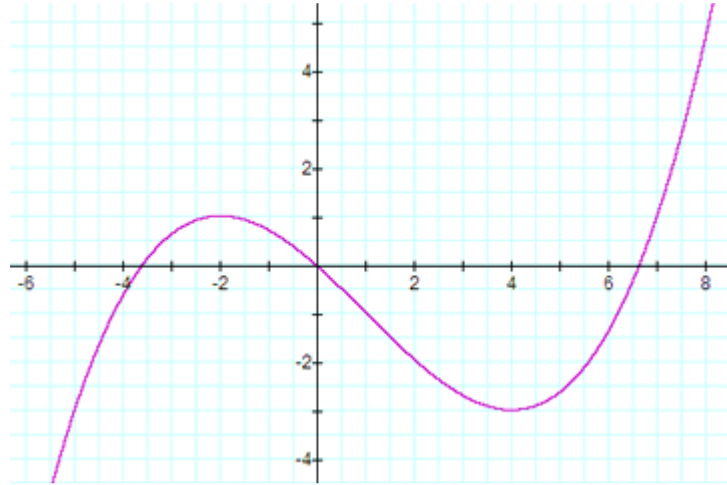
    A.   No solution
    B.   7
    C.   2
    D.   7, 2

3. Determine the interval(s) on which the function is decreasing.   3. _____

A. $(-\infty, -2)$ and $(4, \infty)$

B. $(-\infty, -3.6)$ and $(0, 6.7)$

C. $(-3, 1)$

D. $(-2, 4)$

4. Determine whether the graph of $7+|x|$ is symmetric with respect to the origin,
the x-axis, or the y-axis.   4. _____

A.   not symmetric with respect to the x-axis, not symmetric with respect to the y-axis, and not symmetric with respect to the origin
B.   symmetric with respect to the origin only
C.   symmetric with respect to the x-axis only
D.   symmetric with respect to the y-axis only

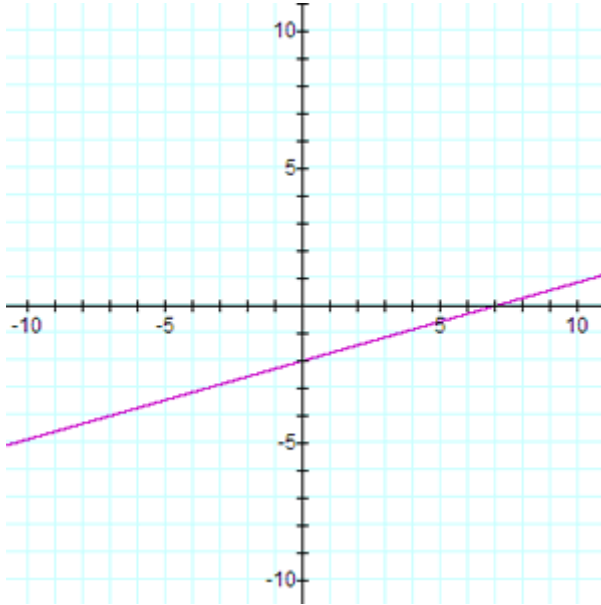5. Solve, and express the answer in interval notation: $|6 - 5x| \geq 14$.   5. _____

A. $(-\infty, -8/5] \cup [4, \infty)$
B. $(-\infty, 4] \cup [-8/5, \infty)$
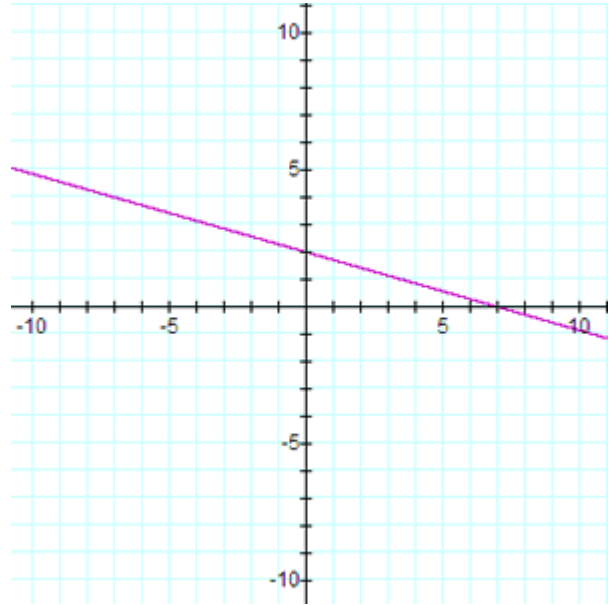C. $[-8/5, \infty)$
D. $[-8/5, 4]$

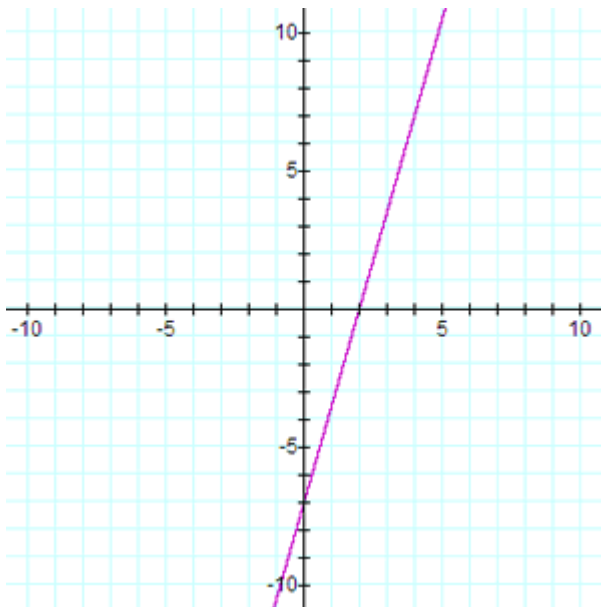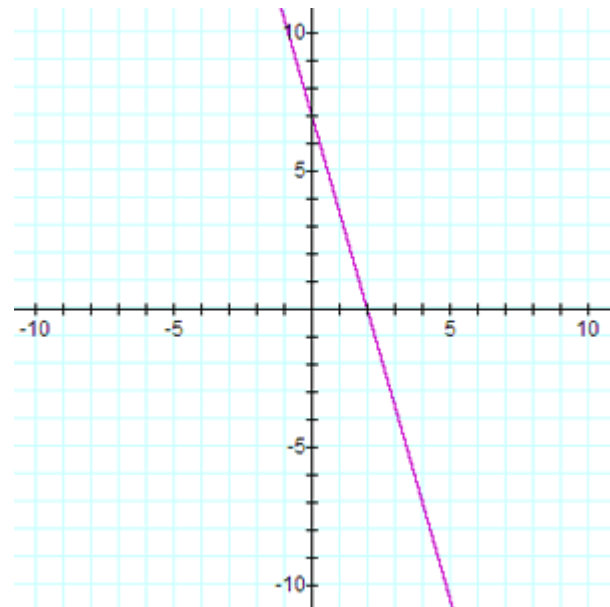6. Which of the following represents the graph of $2x - 7y = 14$?

6. _____

A.

B.

C.

D.

7. Write a slope-intercept equation for a line parallel to the line $x - 7y = 9$ which passes through the point (14, –8).   7. _____

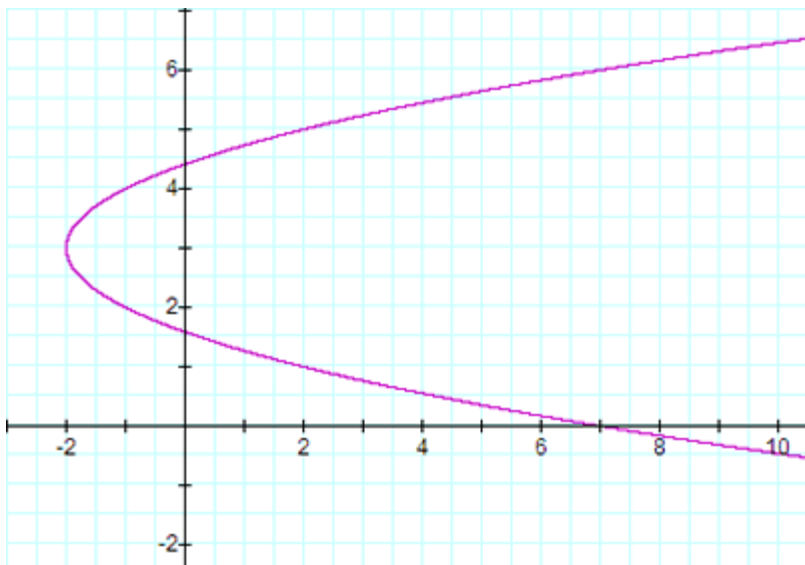   A. $y = -7x + 90$

    B. $y = \dfrac{1}{7}x - 8$

    C. $y = \dfrac{1}{7}x - 10$

    D. $y = -\dfrac{1}{7}x - 6$

8. Which of the following best describes the graph?   8. _____

    A. It is the graph of a function and it is one-to-one.
    B. It is the graph of a function but not one-to-one.
    C. It is the graph of an absolute value relation.
    D. It is not the graph of a function.

9. Express as an equivalent expression: $4 \log y - \log (x^2 +3) + \log 1$ 9. _____
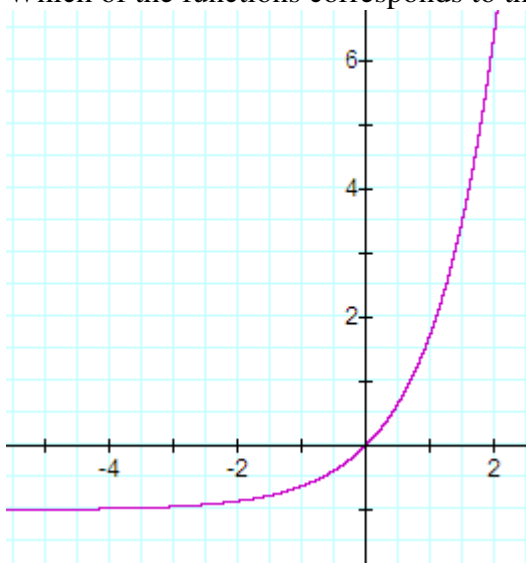
   A. $\dfrac{\log(4y)}{8\log(x)}$

   B. $\log\left(4y - x^2 - 2\right)$

   C. $\log\left(\dfrac{y^4}{x^2 +3}\right)$

   D. $\log\left(\dfrac{y^4}{3x^2}\right)$

10. Which of the functions corresponds to the graph?   10. _____

   A. $f(x) = e^x - 1$
   B. $f(x) = 2e^x - 1$
   C. $f(x) = e^{-x} + 1$
   D. $f(x) = -e^x + 1$

11. Suppose that a function f has exactly two x-intercepts.
    Which of the following statements MUST be true?   11. _____

    A. f is a linear function.
    B.   The equation $f(x) = 0$ has exactly two real-number solutions.
    C. f is a quadratic function.
    D. f is an invertible function.

12. The graph of $y = f(x)$ is shown at the left and the graph of $y = g(x)$ is shown at the right. (No formulas are given.) What is the relationship between $g(x)$ and $f(x)$?

        12. _____
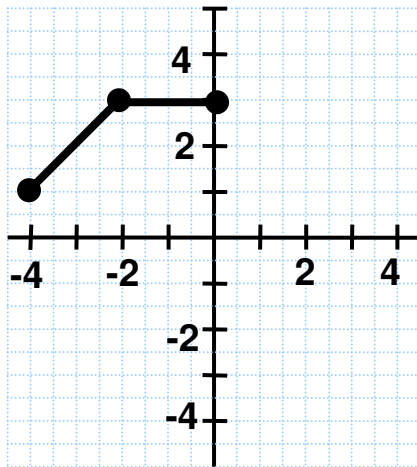
$$y = f(x)$$
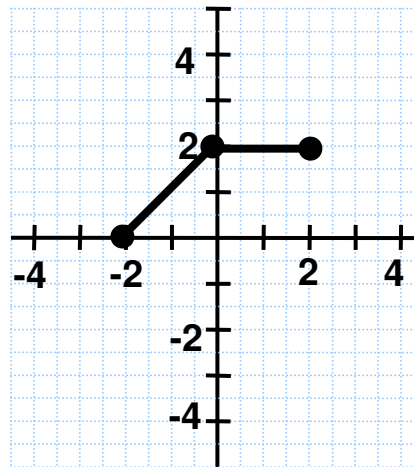
$$y = g(x)$$

    A. $g(x) = f(x + 2) - 1$
    B. $g(x) = f(x - 2) - 1$
    C. $g(x) = f(x + 2) + 2$
    D. $g(x) = f(x + 1) - 2$

SHORT ANSWER:

13. Multiply and simplify: $(4 + 9i)(3 + 2i)$.
    Write the answer in the form $a + bi$, where a and b are real numbers.   Answer: _____

_____

14. Solve, and write the answer in interval notation: $\dfrac{x+2}{x-5} \geq 0$.   Answer: _____

_____

15. A can of soda at 78° F. is placed in a refrigerator that maintains a constant temperature of 38° F. The temperature T of the soda t minutes after it is placed in the refrigerator is given by

$$T(t) = 38 + 40\, e^{-0.058\, t}$$

Find the temperature of the soda 10 minutes after it is placed in the refrigerator. (Round to the nearest tenth of a degree.)

                                                                 Answer: _____

_____

16. Find the value of the logarithm: $\log_2\left(\dfrac{1}{8}\right)$.   Answer: _____

_____

17. Solve: $7^{4x-3} = 49$.   Answer: _____

_____

18. Suppose $7,200 is invested in an account at an annual interest rate of 5.6% compounded continuously. How long (to the nearest tenth of a year) will it take the investment to double in size?   Answer: _____

_____

19. Let $f(x) = x^2 + 10x + 27$.

    (a) Find the vertex.   Answer: _____

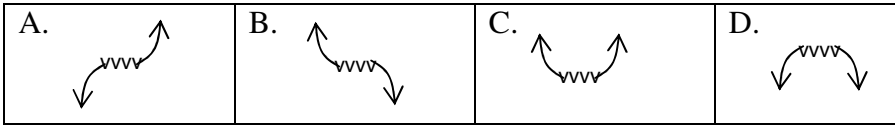   (b) State the range of the function.   Answer: _____

    (c) On what interval is the function decreasing?   Answer: _____

_____

20. Consider the polynomial $P(x)$, shown in both standard form and factored form.

$$P(x) = -\frac{1}{5}x^3 + \frac{1}{5}x^2 + \frac{17}{5}x + 3 = -\frac{1}{5}(x+3)(x+1)(x-5)$$

(a) Which sketch illustrates the end behavior of the polynomial function?

A.	B.	C.	D.

Answer: _____

(b) State the y-intercept.   Answer: _____
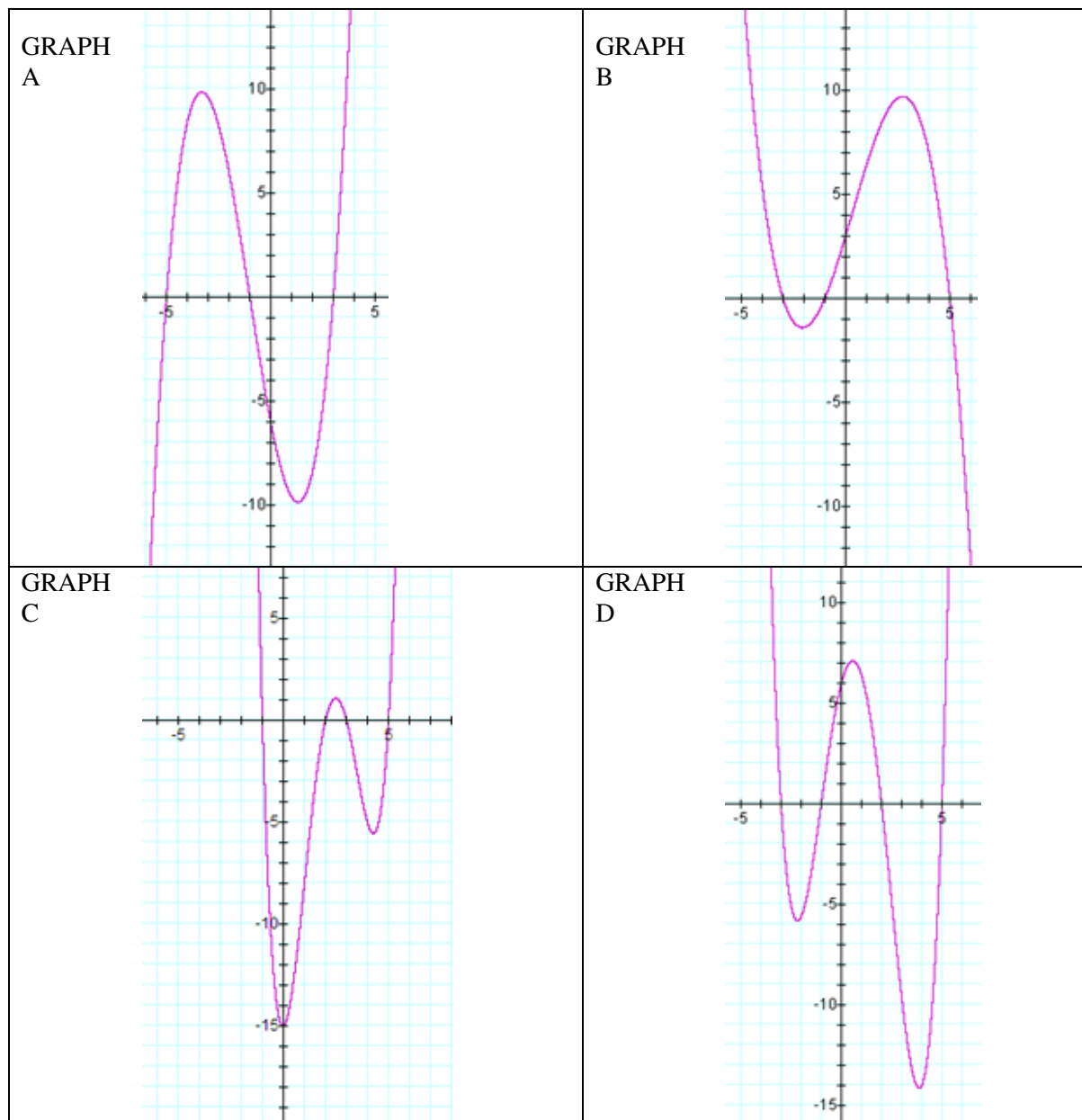
(c) State the zeros of the function.   Answer: _____

(d) State which graph below is the graph of $P(x)$.   Answer: _____

GRAPH A

GRAPH B

GRAPH C

GRAPH D

21. Let $f(x) = \dfrac{3x-2}{x-2}$.

    (a) State the domain.   Answer: _____

    (b) State the horizontal asymptote.   Answer: _____
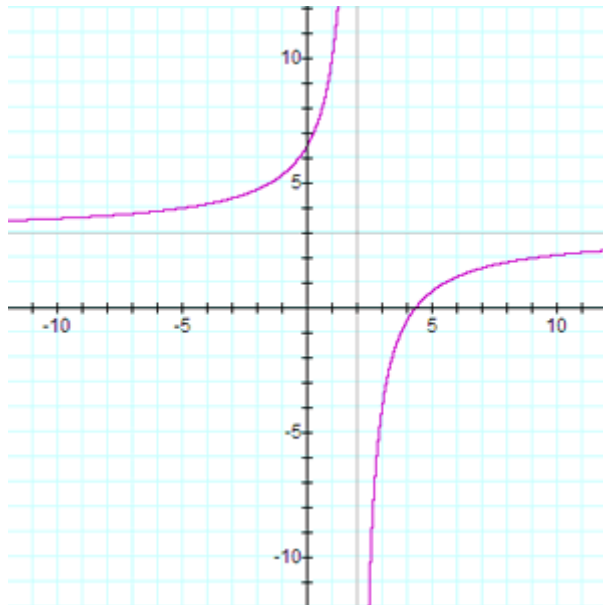
    (c) State the vertical asymptote(s).   Answer: _____
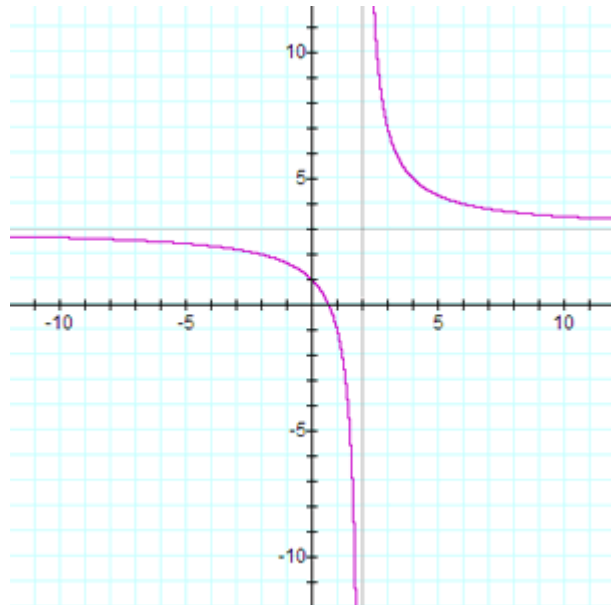
(d) Which of the following represents the graph of $f(x) = \dfrac{3x-2}{x-2}$?   Answer: _____

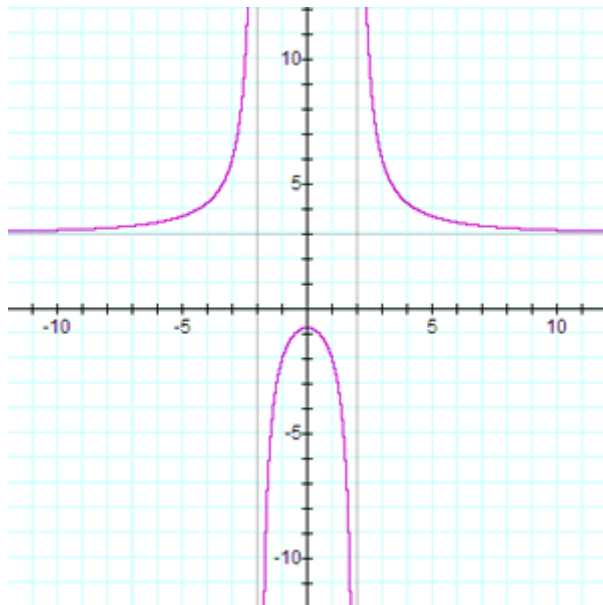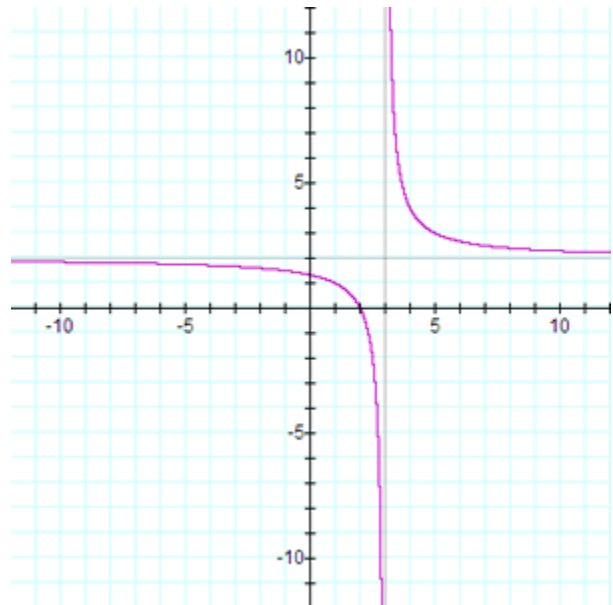GRAPH A.   GRAPH B.

GRAPH C.   GRAPH D.

SHORT ANSWER, with work required to be shown, as indicated.

22. Let $f(x) = x + 5$ and $g(x) = \sqrt{8-x}$.

(a) Find $\left(\dfrac{f}{g}\right)(-1)$. **Show work.**

(b) Find the domain of the quotient function $\dfrac{f}{g}$. **Explain.**

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23. Points (–9, 7) and (–3, 1) are endpoints of the diameter of a circle.

(a) What is the length of the diameter? Give the exact answer, simplified as much as possible. **Show work.**
(b) What is the center point C of the circle?
(c) Given the point C you found in part (b), state the point symmetric to C about the y-axis.

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24. Find the equation for a line which passes through the points (– 4, 8) and (–1, 2). Write the equation in slope-intercept form. **Show work.**

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25. Dan, a resident of Metropolis, pays Metropolis an annual tax of $58 plus 1.9% of his annual income. If Dan paid $1,236 in tax, what was Dan's income? **Show work.**

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26. Let $f(x) = 5x^2 - 9$ and $g(x) = x - 4$.

(a) Find the composite function $(f \circ g)(x)$ and simplify. **Show work.**
(b) Find $(f \circ g)(-1)$. **Show work.**

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27. Find the exact solutions and simplify as much as possible: $2x^2 = 4x + 3$. **Show work.**

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28. Given the function $f(x) = 5 + \dfrac{1}{7}x$, find a formula for the inverse function. **Show work.**

29. The fuel efficiency F for a particular car is given by

$$F(x) = -0.016\, x^2 + 1.28\, x + 12.5$$

where x is the speed of the car in miles per hour (mph) and $F(x)$ is the corresponding fuel efficiency in miles per gallon (mpg).

(a) What is the fuel efficiency if the car's speed is 70 mph?

(b) What speed will yield the maximum fuel efficiency? **Show work.**

30. Solve: $\dfrac{x+8}{x+7} + \dfrac{14}{x^2-49} = 0$. **Show work.**