## Econ 301 Homework 4 <br> due April 4, 2012

Provide complete work that leads to each answer. Do the problems in the order given.

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1. TRUE or FALSE (3 points each). All credits will be assigned to explanations.
a. If a good is a normal good for all consumers, the marginal revenue curve always lies below the demand curve. (Hint: use what we learned in Chapters 8 and 15.)
b. In a two-good model, if one good is an inferior good, then the other good must be a luxury good.
c. Montreal has had a very mild winter and suppose supply of down coats has not changed, then in the market equilibrium, the quantity of down coats will go down because demand decreases.
d. Two firms employ the same factors of production to produce the same product. We also know that both their technologies exhibit constant returns to scale. Then, if the factors firm 1 uses are exactly twice the amount of those firm 2 uses, firm 1 must produce twice the output that firm 2 produces.
2. (8 points) Suppose the market demand for a good consists of two consumers, 1 and 2 , where their respective individual demands are: $D_{1}(p)=200-4 p$ and $D_{2}(p)=100-p$. Note that demand cannot be negative, so what we really mean is $D_{1}(p)=\max \{200-4 p, 0\}$ and $D_{2}(p)=\max \{100-p, 0\}$.
a. (3 points) On one diagram graph the individual demand curves as well as the resulting market demand curve. Mark the intercepts correctly.
b. (2 points) Suppose supply is given by $p=40$, that is, the supply is perfectly elastic. Find the amount purchased by each consumer. Illustrate your answer with a graph.
c. (3 points) Now, suppose instead the supply is given by $q=20$. Find the equilibrium market price. (Be very careful here.) Illustrate your answer with a graph.
3. (6 points) Calculate the income elasticities for the following demand functions.
a. (3 points) $x(p, m)=\frac{m}{2 p}$.
b. (3 points) $x(p, m)=\frac{10}{p}$.
4. (12 points) Suppose the market demand function is given by $D\left(P_{D}\right)=$ $60-P_{D}$ and $S\left(P_{S}\right)=2 P_{S}$.
a. (2 points) Find the market equilibrium without taxes (that is, when $P_{D}=P_{S}$ ).
b. (3 points) Suppose now the supplier is required to play a per unit tax of $t=3$. Draw a graph to show the change in the supply curve. Find the equilibrium quantity and prices, $q^{*}, P_{S}^{*}$, and $P_{D}^{*}$. How much of the per unit tax is passed on to the consumer? (Hint: in this case, the vertical axis is the demand price.)
c. (2 points) Suppose now the demander is required to play a per unit tax of $t=3$. Draw a graph to show the change in the demand curve. Find the equilibrium quantity and prices, $q^{*}, P_{S}^{*}$, and $P_{D}^{*}$. (Hint: in this case, the vertical axis is the supply price.) Compare your answers in parts b and c.
d. (5 points) Calculate the change in consumer's surplus, change in producer's surplus, tax revenue, and deadweight loss as a result of the policy change. Illustrate them on a graph.
5. (10 points) Calculate the marginal products and technical rates of substitution for each of the following technologies. Determine also whether they exhibit increasing, constant, or decreasing returns to scale.
(a) $f\left(x_{1}, x_{2}\right)=x_{1}+2 x_{2}+1$.
(b) $f\left(x_{1}, x_{2}\right)=A x_{1}^{1 / 2} x_{2}^{1 / 3}$.
6. (12 points) Consider a firm that employs capital $k$ and labor $l$ in production. Its production function is $f(k, l)=2 k^{1 / 3} l^{1 / 2}$. The price of output is $p=1$.
a. (3 points) Does the technology exhibit constant, decreasing, or increasing returns to scale?
b. (4 points) Find the marginal product of each factor and the technical rate of substitution. Determine if the technology has diminishing marginal product in either factor.
c. (5 points) Suppose in the short run, capital is fixed at $\bar{k}=27$. What extra information do you need to determine the exact amount of labor to be employed in order to maximize short-run profits? Provide a suitable number yourself, and find how much labor is employed. Illustrate your answer with a graph.
