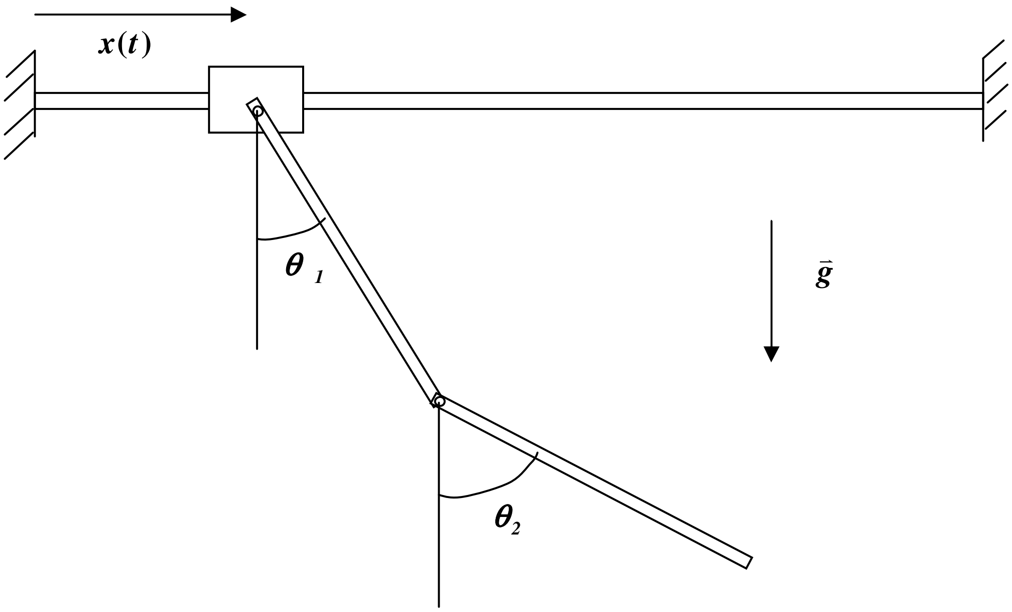
Project #2
MECH 3010: Elementary Numerical Methods and Programming
Fall 2016

Assigned: 11/16/16 **Due: 12/05/16**



This project involes determining the dynamics of the double pendulum with a sliding base (see figure above). Each link is assumed to be of square cross section.

The objective is to determine the angles $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$ over the time period of t = 0 s to t = 4 s. Let the base motion be prescribed as $x(t)=\frac{1}{8}\sin(4.4t)$. Based on the Newton's second law, the equations of motion (for the angular acceleration of each link) are given by

$$\begin{bmatrix} \left(I_c+\frac{5}{4}ml^2\right) & \frac{1}{2}ml^2\cos(\theta_1-\theta_2) \\ \frac{1}{2}ml^2\cos(\theta_1-\theta_2) & \left(I_c+\frac{1}{4}ml^2\right) \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} = -\begin{Bmatrix} \frac{3}{2}ml\,\ddot{x}\cos(\theta_1)+\frac{1}{2}ml^2\sin(\theta_1-\theta_2)\dot{\theta}_2^2+\frac{3}{2}mgl\sin(\theta_1) \\ \frac{1}{2}ml\,\ddot{x}\cos(\theta_2)-\frac{1}{2}ml^2\sin(\theta_1-\theta_2)\dot{\theta}_1^2+\frac{1}{2}mgl\sin(\theta_2) \end{Bmatrix}$$

where $\boldsymbol{m}$ and $\boldsymbol{l}$ are the mass and length of the links and $\boldsymbol{I_c = \frac{ml^2}{12}}$ is the moment of inertia of each link. Important note: the 'dot' notation over the symbol means the corresponding derivative with respect to time.

The values of various parameters to be used in the calculations are:

g = 9.81 m/s^2; l = 0.50 m; ρ = 6500 kg/m^3 (link density); b = 0.10 m (dimension of square link cross-section); m = l*(b*b)*ρ kg; I_c = m*l*l/12.0 m^3.

The above is a system of two 2nd order ordinary differential equations (ODEs). In order to be able to solve this, first transform them into an equivalent four 1st order ODEs. This can be accomplished as follows: Inverting or solving the system of equations above (using the backslash operator or other techniques in MATLAB) and using the fact that $d\theta_1 / dt = \dot{\theta}_1$, $d\dot{\theta}_1 / dt = \ddot{\theta}_1$ (and similarly for θ_2) will provide the values needed to complete the right hand side of the following representation of the system of four 1st order ODEs

$$\dot{\vec{y}} = \vec{f}(t, \vec{y})$$

where

$$\dot{\vec{y}} = \begin{Bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{Bmatrix} \quad \text{and} \quad \vec{y} = \begin{Bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{Bmatrix}$$

Assume that the initial conditions are given by

$$\vec{y}_0 = \begin{Bmatrix} \theta_1(0) \\ \dot{\theta}_1(0) \\ \theta_2(0) \\ \dot{\theta}_2(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Write a MATLAB program that solves for $\vec{y}$ over the interval t = 0 s to t = 4 s using three different methods: the Euler method, the mid-point method (2nd Order Runge-Kutta), and the classical 4th Order Runge-Kutta method.

To evaluate the effect of your step size (h) on the results for each of the three methods, use the following six values of h (units of seconds): 0.02, 0.01, 0.005, 0.0025, 0.00125, and 0.000625. For step sizes 0.01s and smaller, calculate the approximate percent relative error in $\boldsymbol{\theta_1}$ at t = 4 s between the current step size and the next largest step size.

For example, for a step size of 0.01, the approximate percent relative error is:

$$\varepsilon_a(0.01) = \left| \frac{(\theta_1(t = 4s) \text{ using } h = 0.01) - (\theta_1(t = 4s) \text{ using } h = 0.02)}{\theta_1(t = 4s) \text{ using } h = 0.01} \right| * 100\%$$

Turn in a report that contains the following:

1. Title page, including the title of the project and your name
2. Introduction and objectives section:
   - Summarize the problem and state the goals, including what you are trying to find, the ranges of values of t and h to be used, etc.
3. Program design section
   - Describe the overall structure of your program
   - If you used any portions of code from another source (for example, a textbook), cite the source and describe any modifications you made to the code.
   - Describe how you solved for $\ddot{\boldsymbol{\theta}}_1$ and $\ddot{\boldsymbol{\theta}}_2$ in the system of equations (backslash operator, inv(A), or others)
   - Describe how you solved for $\vec{y}$ using the specified values of h

4. Plots. Your report must contain the following five figures with titles, properly labeled axes, and legends where appropriate:
   - Figure 1: On a single graph, plot the approximate percent relative error in θ_1 at t = 4 s vs. step size h for the three different methods.
   - Figure 2: $\boldsymbol{\theta_1}$ vs. t using the largest value of h (0.02 s)
   - Figure 3: $\boldsymbol{\theta_2}$ vs. t using the largest value of h (0.02 s)
   - Figure 4: $\boldsymbol{\theta_1}$ vs. t using the smallest value of h (0.000625 s)
   - Figure 5**:** $\boldsymbol{\theta_2}$ vs. t using the smallest value of h (0.000625 s)

5. Discussion and Conclusions
   - Based on Figure 1, what can you learn about the effects of h on the results obtained with the three different methods?
   - What can you learn by comparing Figure 2 to Figure 4?
   - What can you learn by comparing Figure 3 to Figure 5?
   - If you had to design this system, which differential equation solution method would you use and why?

6. A printout of your code