STAT 462

Homework 5

due Thursday, October 11, 2012

1) Verify the following identity regarding the gamma function: $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$.

2) The length of time until the breakdown of an essential piece of equipment is important in the decision of the use of auxiliary equipment. Assume time to breakdown of a randomly chosen generator, *Y*, follows an exponential distribution with a mean of 15 days.

   (a) What is the probability a generator will break down in the next 21 days?

   (b) A company owns 7 such generators. Let *X* denote the random variable describing how many generators break down in the next 21 days. Assuming the breakdown of any one generator is independent of breakdowns of the other generators, what is the probability that at least 6 of the 7 generators will operate for the next 21 days without a breakdown?

3) Consider the following game: A player throws a fair die repeatedly until s/he rolls a 2, 3, 4, 5 or 6. In other words, the player continues to throw the die as long as s/he rolls 1s. When s/he rolls a "non-1," s/he stops.

   (a) What is the probability the player tosses the die exactly three times?

   (b) What is the expected number of rolls needed to obtain the first non-1?

   (c) If the player rolls a non-1 on the first throw, the player is paid $1. Otherwise, the payoff is doubled for each 1 the player rolls before rolling a non-1. Thus, the player is paid $2 if s/he rolls a 1 followed by a non-1; $4 if s/he rolls two 1s followed by a non-1; $8 if s/he rolls three 1s followed by a non-1; etc. In general, if we let Y be the number of throws needed to obtain the first non-1, then the player rolls $(Y-1)$ 1s before rolling his/her first non-1, and s/he is paid 2^{Y-1} dollars. What is the expected amount paid to the player?

   (d) Suppose the game was modified and the player continues to throw the die until s/he rolls three non-1s. What is the probability the player tosses the die less than 5 times?

4) One of four different prizes was randomly put into each box of a cereal. If a family decided to buy this cereal until it obtained at least one of each of the four different prizes, what is the expected number of boxes of cereal that must be purchased? (*Hint*: Let X = # of boxes of cereal needed to get four prizes. Think of *X* as the sum of four random variables. Y_i = # of boxes needed to get a prize you don't already have, *i* = 1, 2, 3, 4.)

5) In a small pond there are 50 fish, 10 of which have been tagged. If a fisherman's catch consists of 7 fish, selected at random and without replacement, what is the probability exactly two tagged fish are caught?

6) The mean number of bacteria colonies of a certain type in samples of polluted water is 2 per cubic centimeter (cm^3).

   (a) If four 1-cm^3 samples are independently selected from this water, find the probability at least one sample will contain one or more bacteria colonies.

   (b) How many 1-cm^3 samples should be selected in order to have a probability of approximately .95 of seeing at least one bacteria colony in at least one of the 1-cm^3 samples?

   (c) If four 1-cm^3 samples are independently selected from this water, find the probability at least two samples will contain one or more bacteria colonies.

7) A *truncated* discrete distribution is one in which a particular class cannot be observed and is eliminated from the sample space. In particular, if X has range 0, 1, 2, and the 0 class cannot be observed (as is usually the case), the 0-*truncated* random variable X_T has pmf

$$P(X_T = x) = \frac{P(X = x)}{P(X > 0)}, x = 1, 2, \ldots.$$

   Find the pmf, mean and variance of the 0-truncated Poisson (λ) random variable.

8) Do children less than a year old recognize the difference between naughty and nice and show show a preference for nice over naughty? In a study reported in the November 2007 issue of *Nature*, researchers investigated whether infants take into account an individual's actions toward others in evaluating that individual as appealing or aversive. In one component of the study, 10-month-old infants were shown a "climber" character (a piece of wood with "google" eyes glued onto it) that could not make it up a hill in two tries. Then they were alternately shown two scenarios for the climber's next try, one where the climber was pushed to the top of the hill by another character ("helper") and one where the climber was pushed back down the hill by another character ("hinderer"). The infant was alternately shown these two scenarios several times. Then the child was presented with both pieces of wood (the helper and the hinderer) and asked to pick one to play with. The researchers found that 14 of the 16 infants chose the helper over the hinderer.

   (a) If infants really do not show any preference for either type of toy, what is the probability 14 or more of the infants would have chosen the helper toy? What does this suggest about infants making their selections based only on random chance?

   (b) Suppose the study had only 8 infants, and 7 out of the 8 chose the helper toy. Does this result constitute more than, less than, or the same amount of evidence as part (a) that infants show a preference for the helper toy? Explain.

• Book Problems: 4.54, 4.74 (a)-(d) & (f)