2014 Semester One | MATHS 108 – Assignment 1 | Due: 4pm, Tuesday, March 25th

Hand your completed assignment into the correct box in the Student Resource Centre (G38, Building 301) before the due date. Please use a Mathematics Department coversheet available from the Student Resource Centre.
Show all working and note that late assignments will not be marked.

This assignment was written by Andrew Stafford (andrew.stafford@auckland.ac.nz). The tutors in the Mathematics Assistance Room (G16, on the Ground Floor of Building 303) can give you help and advice on the concepts covered, but all work submitted must be your own.

1. Given $A = [-3, 8)$, $B = (6, 8]$ and $\mathbb{N}$, (the set of natural numbers) write down,

   (a) $A \cup B$ in interval notation,

   (b) the set $\mathbb{N} \backslash A$. [2 marks]

2 Which of the following define y as a function of x on $\mathbb{R}$. Explain for each why they are/are not functions.

   (a) $4x^3 + y = 6$ [1 mark]

   (b) $x - y - \sqrt{x} = 8$ [1 mark]

   (c) $x = \cos^2 y$ [1 mark]

   (d) $y = \frac{2x+3}{x-1}$ [1 mark]

3. Let $g(x) = \sin(x)$ and $h(x) = \frac{1}{x}$ be defined on their natural domains. State the following, giving the domain for each function using set notation (show all working).

   (a) $\frac{1}{h(x)}$ (b) $(g \circ h)(x)$

   (c) $(h \circ g)(x)$ (d) $h(x)g(x)$ [4 marks]

4. What is the natural domain of $h(x) = \ln\left(3 - \sqrt{x}\right)$ [2 marks]

5. Use the unit circle to demonstrate that $\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$ [4 marks]

6. Sketch a graph of the following function, labeling all discontinuities, axes intercepts and asymptotes: (you must show all your working. Matlab or Excel may be of help in checking your answer).

$$f(x) = \frac{x^3 + 8x^2 + 11x - 20}{2x^3 - x^2 - 26x + 40}$$

[8 marks]

7. Evaluate the following limits,

(a) $\lim_{x\to 0} \dfrac{x^2 - x - 6}{x^2 - 9}$

(b) $\lim_{x\to \infty} \dfrac{3x^2 + 2x}{5x^2 - 4}$

(c) $\lim_{x\to -\infty} \dfrac{\sqrt{x^2 + 4} - 2}{x}$

(d) $\lim_{x\to 2} \left(3x + \sqrt{\dfrac{x-2}{x^2-4}} \right)$

(e) $\lim_{x\to \infty} \dfrac{|x|}{|x| + x}$

[10 marks]

8. (a) Sketch the following piecewise function. (Matlab or Excel may be of help in checking your answer)

$$f(x) = \begin{cases} -\sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } \pi < x < 4 \\ 2x - 9 & \text{if } x \geq 4 \end{cases}$$

(b) Why is this piecewise function not continuous on $\mathbb{R} > 0$? [5 marks]

9. Let *A*, *B*, and *C* be points with co-ordinates (2, –1, 4), (3, 3, 5) and (–2, 0, 1) respectively

Find

(a) the length of the vector $2\overrightarrow{AB} - 3\overrightarrow{BC}$ [2 marks]

(b) the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ [2 marks]

(c) write a vector equation and corresponding parametric equation for the line through points *B* and *C*. [2 marks]

10. If $\boldsymbol{u}$ = (2, –1, 4), $\boldsymbol{v}$ = (3, 3, 5) and $\boldsymbol{w}$ = (*m*, 0, 1), find

(a) a unit vector perpendicular to $\boldsymbol{u}$ and $\boldsymbol{v}$ [2 marks]

(b) the value of *m* which makes $\boldsymbol{v}$ and $\boldsymbol{w}$ orthogonal. [2 marks]

11. Write at least two sentences explaining which of the questions from this assignment you found most challenging and why. [1 mark]