UNIVERSITY OF HUDDERSFIELD

School of Computing and Engineering

LAPLACE TRANSFORMS

DEFINITION

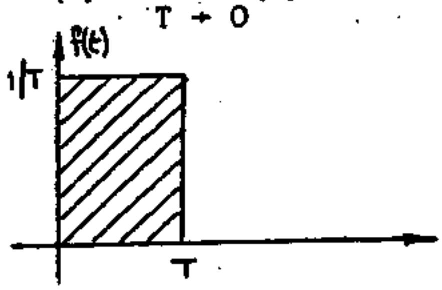
$$F(s) = L.T.[f(t)] = \int_0^{\infty} f(t).e^{-st}.dt$$

OPERATIONS

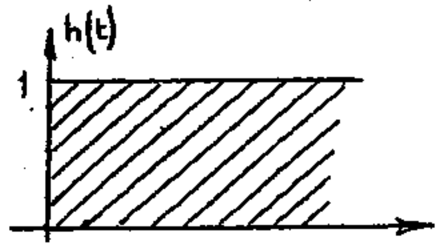
$F(s)$	$f(t)$
$s^nF(s) - s^{n-1}.f(o) - s^{n-2}f^{(1)}(o) - \ldots \ldots - f^{(n-1)}(o)$	$\frac{d^n}{dt^n}.[f(t)]$
$\frac{1}{s}.F(s)$	$\int_0^t f(t).dt$
$\frac{1}{s}.F(s) + \frac{1}{s} f^{(-1)}(o+)$	$\int f(t).dt$
$F(s + \alpha)$	$e^{-\alpha t} f(t)$
$e^{-sT}F(s)$	$f(t - T)$

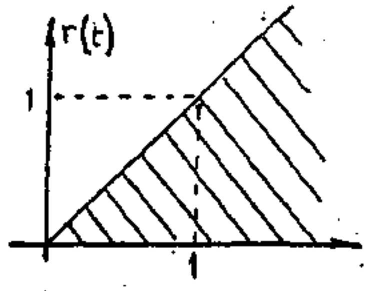
TRANSFER PAIRS

	$F(s)$	$f(t)$, $f(t) = o$ for $t < o$
1.	1	Unit Impulse Function $\delta(t) = \lim_{T \to 0} f(t)$



2.	$\frac{1}{s}$	Unit step function $h(t)$



	F(s)	f(t)	
3.	$\frac{1}{s^2}$	Unit ramp function r(t)	
4.	$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n integral)	$\frac{t^{n-1}}{\Gamma(n)}$ (n non-integral) (Γ - Gamma function)
5.	$\frac{1}{s+a}$		e^{-at}
6.	$\frac{1}{(s+a)^2}$		te^{-at}
7.	$\frac{s}{(s+a)^2}$		$e^{-at}(1-at)$
8.	$\frac{1}{(s+a)^n}$		$\frac{1}{(n-1)!}t^{n-1}e^{-at}$
9.	$\frac{1}{s(s+a)}$		$\frac{1}{a}(1-e^{-at})$
10.	$\frac{1}{s(s+a)^2}$		$\frac{1}{a^2}[1-e^{-at}(1+at)]$
11.	$\frac{1}{s^2(s+a)}$		$\frac{1}{a^2}[at-(1-e^{-at})]$
12.	$\frac{1}{(s+a)(s+b)}$ $a \neq b$		$\frac{1}{(b-a)}[e^{-at}-e^{-bt}]$
13.	$\frac{s}{(s+a)(s+b)}$ $a \neq b$		$\frac{1}{(a-b)}[ae^{-at}-be^{-bt}]$

	F(s)	f(t)
14.	$\frac{1}{s(s+a)(s+b)}$ $a \neq b$	$\frac{1}{ab(a-b)}[be^{-at}-ae^{-bt}]+\frac{1}{ab}$
15.	$\frac{1}{s^2+\omega^2}$	$(\frac{1}{\omega})\sin(\omega t)$
16.	$\frac{s}{s^2+\omega^2}$	$\cos(\omega t)$
17.	$\frac{1}{s(s^2+\omega^2)}$	$(\frac{1}{\omega^2})[1-\cos(\omega t)]$
18.	$\frac{1}{s^2-\beta^2}$	$(\frac{1}{\beta})\sinh(\beta t)$
19.	$\frac{s}{s^2-\beta^2}$	$\cosh(\beta t)$
20.	$\frac{1}{s(s^2-\beta^2)}$	$(\frac{1}{\beta^2})[\cosh(\beta t)-1]$
21.	$\frac{1}{(s+a)(s^2+\omega^2)}$	$\frac{1}{\omega(a^2+\omega^2)}[a\ \sin(\omega t)-\omega\cos(\omega t)+\omega e^{at}]$ or (with simple manipulation) $\frac{1}{\omega\sqrt{(a^2+\omega^2)}}\{\sin(\omega t-\theta)+\sin\theta . e^{-at}\}$, where $\theta = \tan^{-1}(\frac{\omega}{a})$.
22.	$\frac{s}{(s+a)(s^2+\omega^2)}$	$\frac{1}{(a^2+\omega^2)}[a\ \cos(\omega t)+\omega\sin(\omega t)-ae^{-at}]$ or $\frac{1}{\sqrt{(a^2+\omega^2)}}\{\cos(\omega t-\theta)-\frac{a}{\omega}\sin\theta . e^{-at}\}$ where $\theta = \tan^{-}(\frac{\omega}{a})$.

F(s)	f(t)
23. $\frac{1}{s(s+a)(s^2+\omega^2)}$	$(\frac{1}{a\omega^2})\{1-\frac{1}{(a^2+\omega^2)}\cdot[a^2\cos(\omega t)+a\omega\sin(\omega t)+\omega^2 e^{-at}]\}$ or $(\frac{1}{a\omega^2})\{1-\cos\theta.\cos(\omega t-\theta)-\sin^2\theta.e^{-at}\}$, where $\theta = \tan^{-1}(\omega/a)$.
24. $\frac{1}{(s+a)(s^2-\beta^2)}$	$\frac{1}{\beta(a^2-\beta^2)}[a\ \sinh(\beta t)-\beta\cosh(\beta t)+\beta e^{-at}]$
25. $\frac{s}{(s+a)(s^2-\beta^2)}$	$\frac{1}{(a^2-\beta^2)}[a\ \cosh(\beta t)-\beta\sinh(\beta t)-ae^{-at}]$
26. $\frac{1}{s(s+a)(s^2-\beta^2)}$	$\frac{1}{\beta^2}\{\frac{1}{(a^2-\beta^2)}[a^2\cosh(\beta t)-a\beta\sinh(\beta t)-\beta^2 e^{-at}]-1\}$
27. $\frac{1}{s^2+2\alpha s+\omega_o^2}$	$\omega_o > \alpha; \frac{1}{\omega_d}e^{-\alpha t}\sin(\omega_d t)$. where $\omega_d = \sqrt{(\omega_o^2 - \alpha^2)}$; $\omega_o < \alpha; \frac{1}{\beta}e^{-\alpha t}.\sinh(\beta t)$ where $\beta = \sqrt{(\alpha^2-\omega_o^2)}$. $\omega_o = \alpha$; reduces to entry No. 6.
28. $\frac{s}{s^2+2\alpha s+\omega_o^2}$	$\omega_o > \alpha; e^{-\alpha t}[\cos(\omega_d t)-(\frac{\alpha}{\omega_d})\sin(\omega_d t)]$ where $\omega_d = \sqrt{(\omega_o^2-\alpha^2)}$ or $-(\frac{\omega_o}{\omega_d})e^{-\alpha t}\sin(\omega_d t-\theta)$ where $\theta = \tan^{-1}(\omega_d/\alpha)$; $\omega_o < \alpha; e^{-\alpha t}[\cosh(\beta t)-(\frac{\alpha}{\beta})\sinh(\beta t)]$ where $\beta = \sqrt{(\alpha^2-\omega_o^2)}$; $\omega_o = \alpha$; reduces to entry No. 7.

F(s) f(t)

29. $\dfrac{1}{s(s^2+2\alpha s+\omega_0^2)}$

$\omega_o > \alpha; (\frac{1}{\omega_0^2})[1-e^{-\alpha t}\{\cos(\omega_d t)+(\frac{\alpha}{\omega_d})\sin(\omega_d t)\}]$

where $\omega_d = \sqrt{(\omega_0^2-\alpha^2)}$

or

$(\frac{1}{\omega_0^2})[1-(\frac{\omega_o}{\omega_d})e^{-\alpha t}\sin(\omega_d t+\theta)]$

where $\theta = \tan^{-1}(\omega_d/\alpha)$

$\omega_o < \alpha; (\frac{1}{\omega_0^2})[1-e^{-\alpha t}\{\cosh(\beta t)+(\frac{\alpha}{\beta})\sinh(\beta t)\}]$

where $\beta = \sqrt{(\omega^2-\omega_0^2)}$

$\omega_o = \alpha$; reduces to entry No. 10

---

30. $\dfrac{1}{(s+a)(s+b)(s+c)}$

$$\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(a-b)(c-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$$

31. $\dfrac{s}{(s+a)(s+b)(s+c)}$

$$\frac{ae^{-at}}{(a-b)(c-a)} + \frac{be^{-bt}}{(b-a)(c-b)} + \frac{ce^{-ct}}{(c-a)(b-c)}$$

32. $\dfrac{1}{s(s+a)(s+b)(s+c)}$

$$\frac{1}{abc} - \frac{e^{-at}}{a(b-a)(c-a)} - \frac{e^{-bt}}{b(a-b)(c-b)} - \frac{e^{-ct}}{c(a-c)(b-c)}$$